Applying Parameterized Complexity to Cognitive Science

Ulrike Stege

University Of Victoria, Canada, BC
Acknowledgements

- Sarah Carruthers, Mike Masson
- Mike Fellows, Iris van Rooij
Motivation

- Cognitive Scientists want to get insight into
  - power of cognitive system
  - what do/can we compute?
  - human problem-solving capabilities
- How do we go about it?
Cognitive System

- Cognitive system performs *cognitive tasks*

- Cognitive task
  - corresponds to computational task (Massaro & Cowan, 1993)
  - modelled / described by *cognitive function*

- *cognitive process* is the mechanism that realizes the cognitive function in the cognitive system
We are interested in

- Plausible cognitive functions
- Plausible cognitive processes computing these functions
Assumption (Church-Turing): All cognitive functions are computable
David Marr’s Level Theory
[Marr, 1982]

- Widely accepted distinction between
  - computational level
  - algorithmic level
  - implementation level
David Marr’s Level Theory
[Marr, 1982]

- Our goals concentrate on two upper levels. Contribute to
  - determine computational level theories (i.e., cognitive functions)
  - find algorithmic level theories that explain given computational level theories
    - i.e., determine an algorithm that people use to solve any instance for a given computational problem/cognitive function
Determining Cognitive Functions

- Assumption: cognitive functions are computable
- What other properties must cognitive functions have?
  - some kind of tractability
The Tractable Cognition Thesis
[Frixione, 2001; van Rooij 2003; 2008]

Set of all computable functions
Cognitive Functions: Tractability

- Assumption: cognitive functions are computable
- What other properties must cognitive functions have?
  - some kind of tractability
- Can, e.g., NP-hard computational problems be plausible models for cognitive functions?
Do people solve computationally hard problems?

Functional Characterizations of Cognitive Tasks

- Examples
  - visual perception [Tsotsos, 1988; 1989;1990; 1991]
  - analogical reasoning [Thagard, 2000]
  - linguistic processing [Wareham, 1996,1998]
  - decision making [van Rooij, S., Kadlec, 2005]

Suggested computational problems as models for these functions all NP-hard
Do people solve computationally hard problems? Experimental Evidence

• People are good at solving Euclidean Traveling Salesman instances
  [Graham, Joshi, & Pizlo, 2000; Ormerod & Chronicle, 1999; van Rooij, S. et al., 2006; Walwyn & Navarro, 2011]

• taking roughly linear time for delivering good solutions
  [Graham, Joshi, & Pizlo, 2000; Dry et al., 2006; Dry, Preiss & Wagemans, 2012]

• Interesting hierarchical pyramid model suggested
  [Graham, Joshi, & Pizlo, 2000; Haxhimusa et al. 2006]
Do people solve computationally hard problems? What we like to do ....

Many people LOVE solving instances of computationally hard problems!!

[Image of a grid with numbers and a chart showing computational complexity]

classically intractable
Computational Classical Intractability = Cognitive Intractability?

- What to do when cognitive function is, e.g., NP-hard
  - Heuristic explanation on algorithmic level
  - Reject the function from the plausible ones
  - Refined analysis
Psychological Explanations: Intractability and the Use of Heuristics

- A number of cognitive scientists suggest that intractable cognitive functions invoke heuristics as algorithmic level explanation (e.g., [Millgram, 2000; Thagard, 1998])
- Rejected as plausible explanation [van Rooij, Wright, Wareham, 2012]
- Heuristics compute different functions, not equal to the cognitive function assumed to model the cognitive task
P-Cognition Thesis [Frixione, 2001]
FPT-Cognition Thesis [van Rooij, 2003; 2008]
Parameterized Complexity Studies for Cognitive Functions

- Fixed-parameter tractable parameterizations exist for:
  - Analogical Reasoning: Coherence [van Rooij, 2003]
  - Decision Making: Subset Choice [van Rooij, S., Kadlec, 2005]
  - Linguistic Processing: Declarative Phonology [Wareham 1996; 1999]
Coherence

- **Input**: $G = (P, C)$ with vertex or *proposition* set $P$ and edge or *constraint* set $C$ consisting of positive and negative constraints (that is $C$ is partitioned into $C^+$ and $C^-$, edge weights $w(pq)$ for each $pq \in C$, integer $c > 0$

- **Question**: Does there exist a partition of $P$ into sets $A$ and $R$ s.t.
  $$\sum_{pq \in S_G(A,R)} w(pq) \geq c ?$$

- **$S_G(A, R)$**: Subnetwork that contains
  all edges $pq \in C^+$ with either $p, q \in A$ or $p, q \in R$ and
  all edges $pq \in C^-$ with either $p \in A$ and $q \in R$ or vice versa
Coherence as Cognitive Theory

- Coherence was suggested to model cognitive tasks in various domains. Examples [Thagard & others]
  - scientific explanation
  - legal justification
  - social judgment
  - visual perception
Paul Thagard to Iris van Rooij in 2007:

“[...] Hence the natural formal generalization is your $|C^-|$-coherence, which you've shown to be FPT. This still leaves open the question of what algorithm for computing this restricted kind of coherence is most psychologically plausible. [...]”
Algorithmic Level: Human Problem Solving of (NP-Hard) Computational Problems

- Experimental studies
- E-TSP
- Vertex Cover [Carruthers, Masson, S., 2012]
- Vertex Cover versus Independent Set
Experimental Studies: Goals, Some Challenges

- Human problem solving on (hard) computational problems: What do we want?
  - study the human performance (solution quality)
  - determine the strategies used when problem solving
  - do human strategies make use of tractable parameterizations?
  - do human strategies make use of P-time reduction rules?
Experimental Studies: Goals, Challenges

- What are the limitations?
  - instructions
  - presentation (e.g., graph vs. matrix)
  - representation (e.g., graph layout)
  - time constraints
  - instance selection
  - feedback/cognitive support
Human Problem Solving: Vertex Cover & Independent Set

- classically equivalent
- natural parameterizations
  - $k$-Vertex Cover in FPT
  - $k$-Independent Set $W[1]$-complete
Independent Set vs. Vertex Cover

- natural parameterization for Vertex Cover: in FPT
- natural parameterization for Independent Set: W[1]-complete
- Given $G = (V, E)$. $G$ has a vertex cover ($vc$) of size at most $k$ iff $G$ has an independent set ($is$) of size at least $|V| - k$. 
Independent Set vs. Vertex Cover

* natural parameterization for Vertex Cover: in FPT
* natural parameterization for Independent Set: W[1]-complete

Given $G = (V,E)$. $G$ has a vertex cover ($vc$) of size at most $k$ iff $G$ has a independent set ($is$) of size at least $|V|-k$. 

optimal $vc$
Independent Set vs. Vertex Cover

- natural parameterization for Vertex Cover: in FPT
- natural parameterization for Independent Set: \( W[1] \)-complete
- Given \( G = (V,E) \). \( G \) has a vertex cover \((vc)\) of size at most \( k \) iff \( G \) has a independent set \((is)\) of size at least \(|V|-k\).
Vertex Cover—Participant Instructions

- Graph as art gallery (rooms = nodes, corridors = edges); optimization version
  - Find the fewest guards necessary to protect the art in the corridors of the art gallery
Independent Set—Instructions

- Social network: Independent people problem; optimization version
Reduction Rules studied

RR1
Reduction Rules studied

RR1

optimal $vc$ selection
Reduction Rules studied

RR1

- optimal \( vc \) selection
- optimal \( is \) selection
Reduction Rules studied

RR1

optimal \( vc \) selection

optimal \( is \) selection

RR2
Reduction Rules studied

RR1
- optimal \( vc \) selection
- optimal \( is \) selection

RR2
Reduction Rules studied

RR1
- optimal $\nu c$ selection
- optimal $is$ selection

RR2
Vertex Cover—Reduction Rule
Recognition by Participants

- Experiment 1: 14 participants (non-CS/MATH), 24 instances [Carruthers, Masson, S., 2012]; Vertex Cover only
- Experiment 2: 16 participants each (non-CS/MATH), 16 instances; Vertex Cover vs. Independent Set

  - RR1
    - highly likely to choose neighbors of pendant vertices to add to vertex cover when available.
    - participants are likely able to identify RR1
Vertex Cover—Reduction Rule Recognition by Participants

- **Experiment 1:** 14 participants (non-CS/MATH), 24 instances [Carruthers, Masson, S., 2012]; Vertex Cover only

- **Experiment 2:** 16 participants each (non-CS/MATH), 16 instances; Vertex Cover vs. Independent Set

- RR2

  - **likely** to choose to add neighbors of degree-2 vertices in triangles where available
Independent Set vs. Vertex Cover: Reduction Rule Selection (Experiment 2)

Proportion Degree 1 and Degree 2 Vertices Selected by Participant

Proportion D1 and D2 Rule Vertices Selected by Participant

Independent Set

Vertex Cover
Solution quality: IS vs. VC

Independent Set Group Mean PAO by Isomorph

Vertex Cover Group Mean PAO by Isomorph
Summary

• In preliminary study

• Solving VC appears easier than IS
  
  • people are better at solving VC than IS

• RR1 picked up on pretty consistently for both VC and IS

• RR2 did much better for VC

• Next step: Investigate reasons why IS is solved not as good
Conclusions and Challenges

• Tractability of cognitive functions

• full array of parameters worth to be studied (suggested by Wareham)

• which parameters are picked up in human problem solving strategies?

• “Equivalent” problem formulations may impact cognitive tractability

• Tractable Cognitions Thesis: Tractable $\subseteq$ FPT may be too narrow (e.g. TSP)

• e.g. visual properties (E-TSP)

• Cognitive tractable $\subseteq$ P-size kernelizable + some other properties?