Applying Parameterized Complexity to Cognitive Science

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Motivation

- Cognitive Scientists want to get insight into
  - power of *cognitive system*
    - what do/can we compute?
  - human problem-solving capabilities
    - How do we go about it?

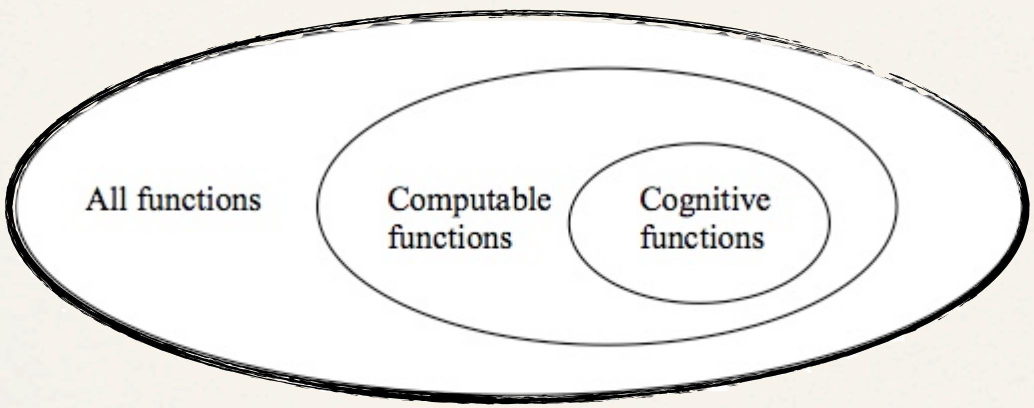
Cognitive System

- Cognitive system performs *cognitive tasks*
- Cognitive task
  - corresponds to computational task (Massaro & Cowan, 1993)
  - modelled / described by *cognitive function*
- *cognitive process* is the mechanism that realizes the cognitive function in the cognitive system

We are interested in

- Plausible cognitive functions
- Plausible cognitive processes computing these functions

Assumption (Church-Turing): All cognitive functions are computable



David Marr's Level Theory [Marr, 1982]

- Widely accepted distinction between
  - **computational level**
  - **algorithmic level**
  - implementation level

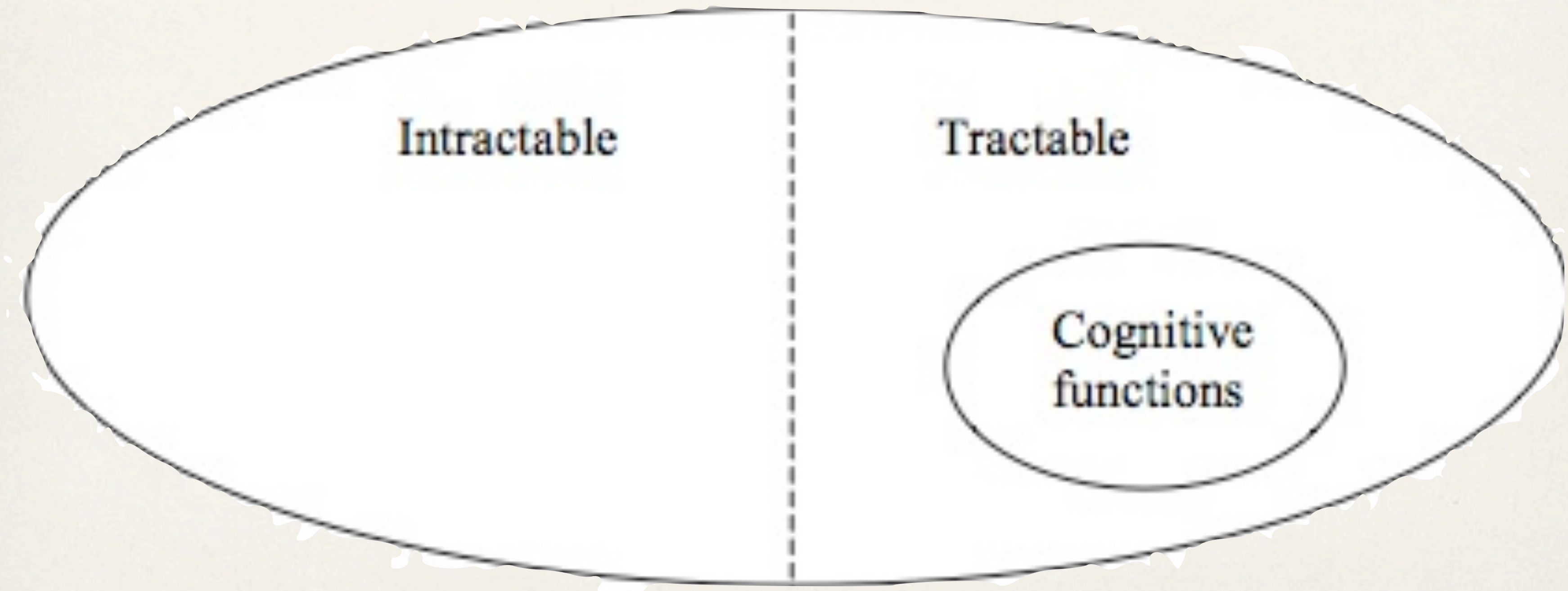
David Marr's Level Theory [Marr, 1982]

- **Our goals concentrate on two upper levels. Contribute to**
  - determine **computational level** theories (i.e., cognitive functions)
  - find **algorithmic level** theories that explain given computational level theories
    - i.e., determine an algorithm that people use to solve any instance for a given computational problem/cognitive function

Determining Cognitive Functions

- Assumption: cognitive functions are computable
- What other properties must cognitive functions have?
    - some kind of tractability

The Tractable Cognition Thesis [Frixione, 2001; van Rooij 2003; 2008]



Set of all computable functions

Cognitive Functions: Tractability

- Assumption: cognitive functions are computable
- What other properties must cognitive functions have?
  - some kind of tractability
- Can, e.g., NP-hard computational problems be plausible models for cognitive functions?

Do people solve computationally hard problems? Functional Characterizations of Cognitive Tasks

- Examples
  - visual perception [Tsotsos, 1988; 1989;1990; 1991]
  - analogical reasoning [Thagard, 2000]
  - linguistic processing [Wareham, 1996,1998]
  - decision making [van Rooij, S., Kadlec, 2005]

suggested computational problems as models for these functions all NP-hard

Do people solve computationally hard problems? Experimental Evidence

- People are good at solving *Euclidean Traveling Salesman* instances

  [Graham, Joshi, & Pizlo, 2000; Ormerod & Chronicle, 1999; van Rooij, S. *et al.*, 2006; Walwyn & Navarro, 2011]

  - taking roughly linear time for delivering good solutions

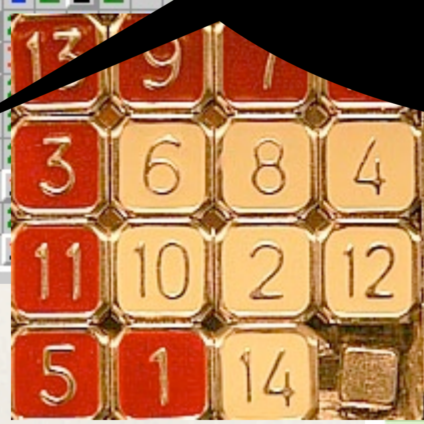
    [Graham, Joshi, & Pizlo, 2000; Dry *et al.*, 2006; Dry, Preiss & Wagemans, 2012]

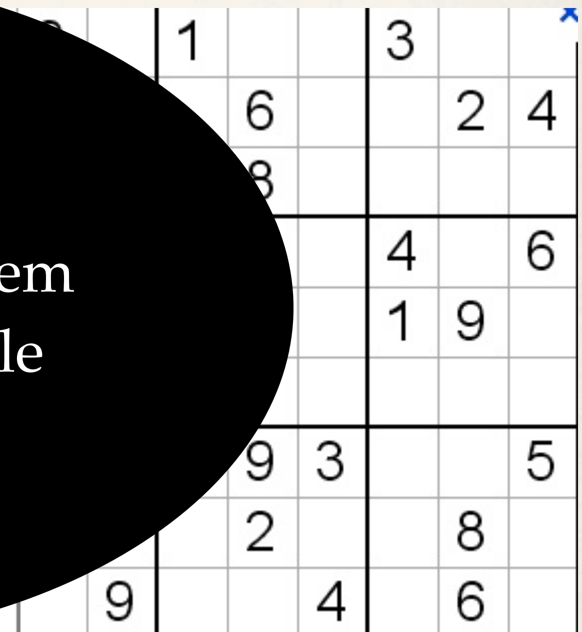
- Interesting hierarchical pyramid model suggested

  [Graham, Joshi, & Pizlo, 2000; Haxhimusa *et al.* 2006]

Do people solve computationally hard problems? What we like to do ....

- Many people LOVE solving
  computationally hard





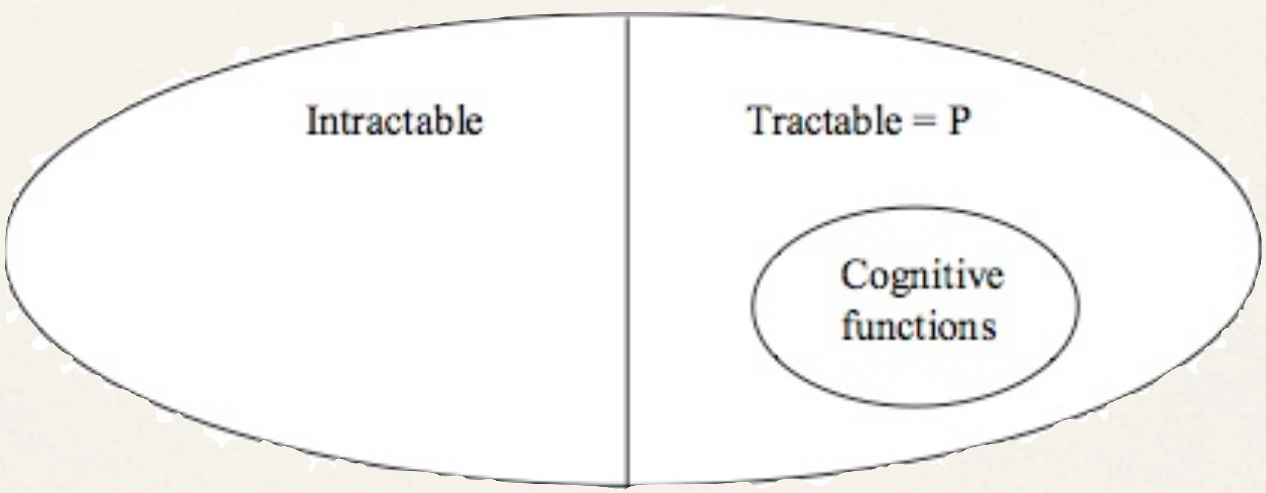
Computational Classical Intractability = Cognitive Intractability?

- What to do when cognitive function is, e.g., NP-hard
  - Heuristic explanation on algorithmic level
  - Reject the function from the plausible ones
  - Refined analysis

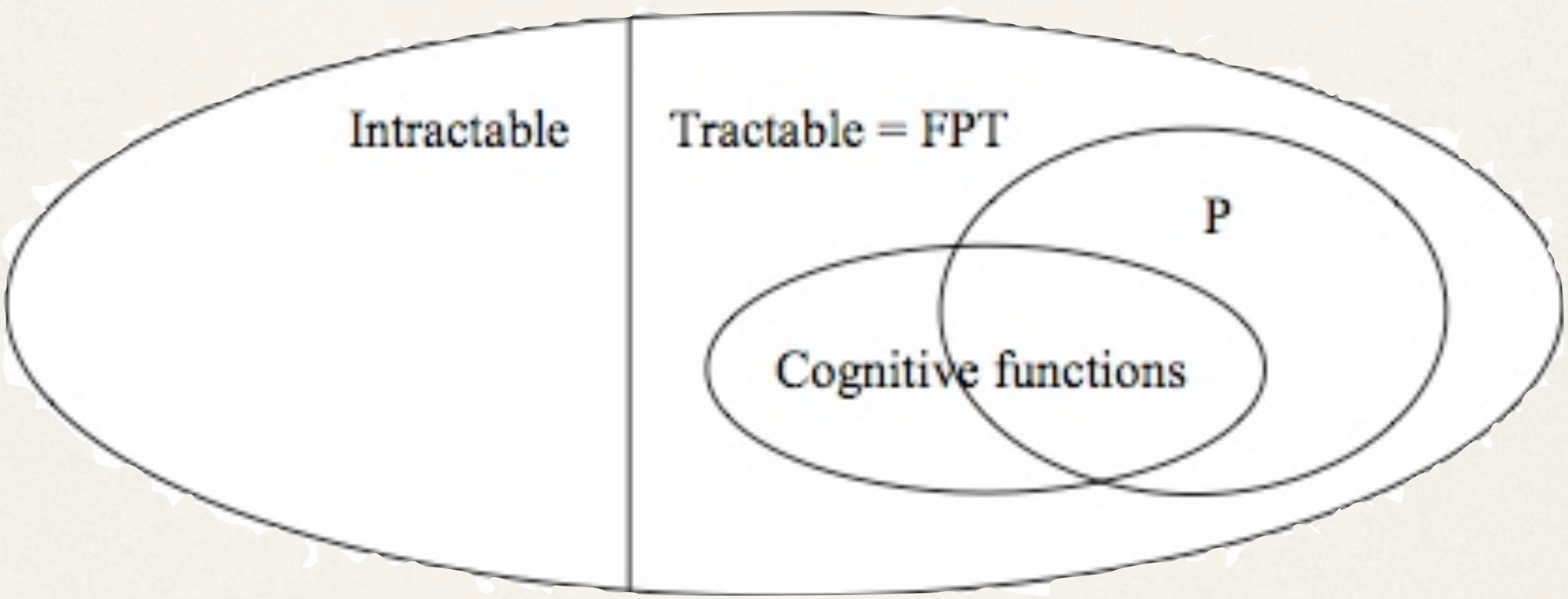
Psychological Explanations: Intractability and the Use of Heuristics

- A number of cognitive scientists suggest that intractable cognitive functions invoke *heuristics* as algorithmic level explanation (e.g., [Millgram, 2000; Thagard, 1998]
- Rejected as plausible explanation [van Rooij, Wright, Wareham, 2012]
  - heuristics compute different functions, not equal to the cognitive function assumed to model the cognitive task

P-Cognition Thesis [Frixione, 2001]



FPT-Cognition Thesis [van Rooij, 2003; 2008]



Parameterized Complexity Studies for Cognitive Functions

- Fixed-parameter tractable parameterizations exist for:
  - Analogical Reasoning: Coherence [van Rooij, 2003]
  - Decision Making: Subset Choice [van Rooij, S., Kadlec, 2005]
  - Linguistic Processing: Declarative Phonology [Wareham 1996; 1999]

Coherence

- **Input**: $G = (P, C)$ with vertex or *proposition* set P and edge or *constraint* set C consisting of positive and negative constraints (that is C is partitioned into C^+ and C^-, edge weights $w(pq)$ for each $pq \in C$, integer $c > 0$

- **Question**: Does there exist a partition of P into sets A and R s.t.
$$\sum_{pq \in S_G(A,R)} w(pq) \geq c \text{ ?}$$

- $S_G(A, R)$: Subnetwork that contains all edges $pq \in C^+$ with either $p, q \in A$ or $p, q \in R$ and all edges $pq \in C^-$ with either $p \in A$ and $q \in R$ or vice versa

Coherence as Cognitive Theory

- Coherence was suggest to model cognitive tasks in various domains. Examples [Thagard & others]
  - scientific explanation
  - legal justification
  - social judgment
  - visual perception

Coherence as Cognitive Theory (2)

- Paul Thagard to Iris van Rooij in 2007:
  - "[....] Hence the natural formal generalization is your $|C^-|$-coherence, which you've shown to be FPT. This still leaves open the question of what algorithm for computing this restricted kind of coherence is most psychologically plausible. [...]"

Algorithmic Level: Human Problem Solving of (NP-Hard) Computational Problems

- Experimental studies
  - E-TSP
  - Vertex Cover [Carruthers, Masson, S., 2012]
- Vertex Cover versus Independent Set

Experimental Studies: Goals, Some Challenges

- Human problem solving on (hard) computational problems: What do we want?
  - study the human performance (solution quality)
  - determine the strategies used when problem solving
  - do human strategies make use of tractable parameterizations?
  - do human strategies make use of P-time reduction rules?

Experimental Studies: Goals, Challenges

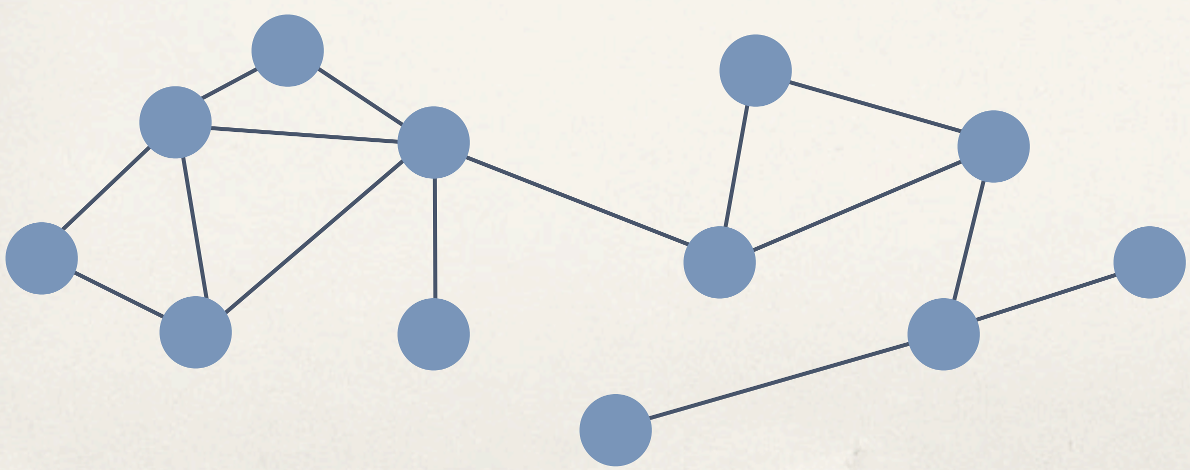
- What are the limitations?
  - instructions
  - presentation (e.g., graph vs. matrix)
  - representation (e.g., graph layout)
  - time constraints
  - instance selection
  - feedback/cognitive support

Human Problem Solving: Vertex Cover & Independent Set

- classically equivalent
- natural parameterizations
  - k-Vertex Cover in FPT
  - k-Independent Set W[1]-complete

Independent Set vs. Vertex Cover

- natural parameterization for Vertex Cover: in FPT
- natural parameterization for Independent Set: W[1]-complete
- Given $G = (V,E)$. G has a vertex cover (*vc*) of size at most k iff G has a independent set (*is*) of size at least $|V|- k$.



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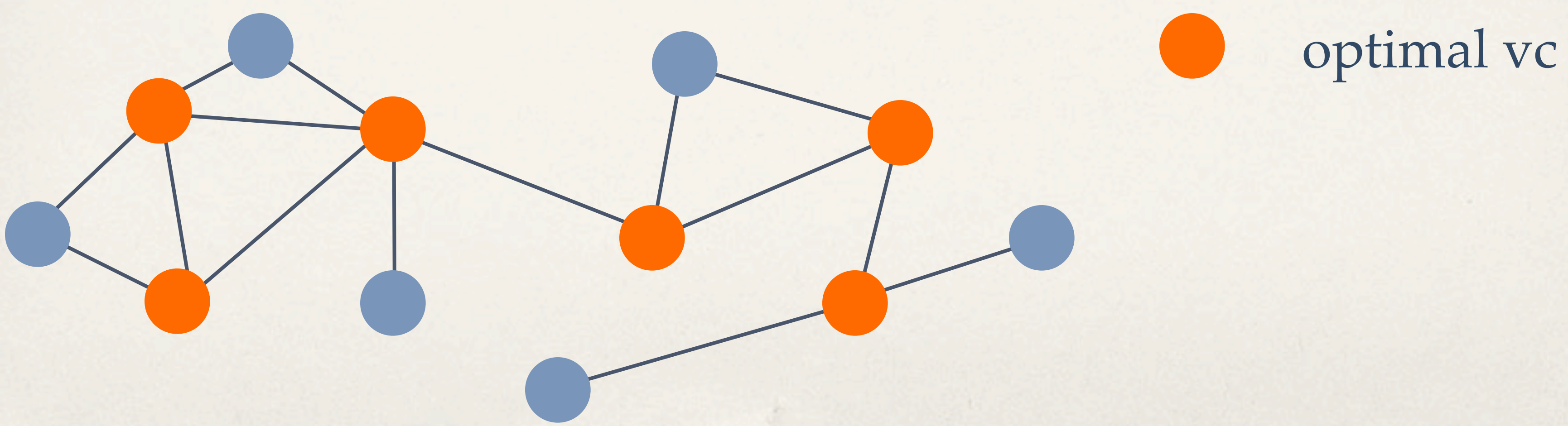

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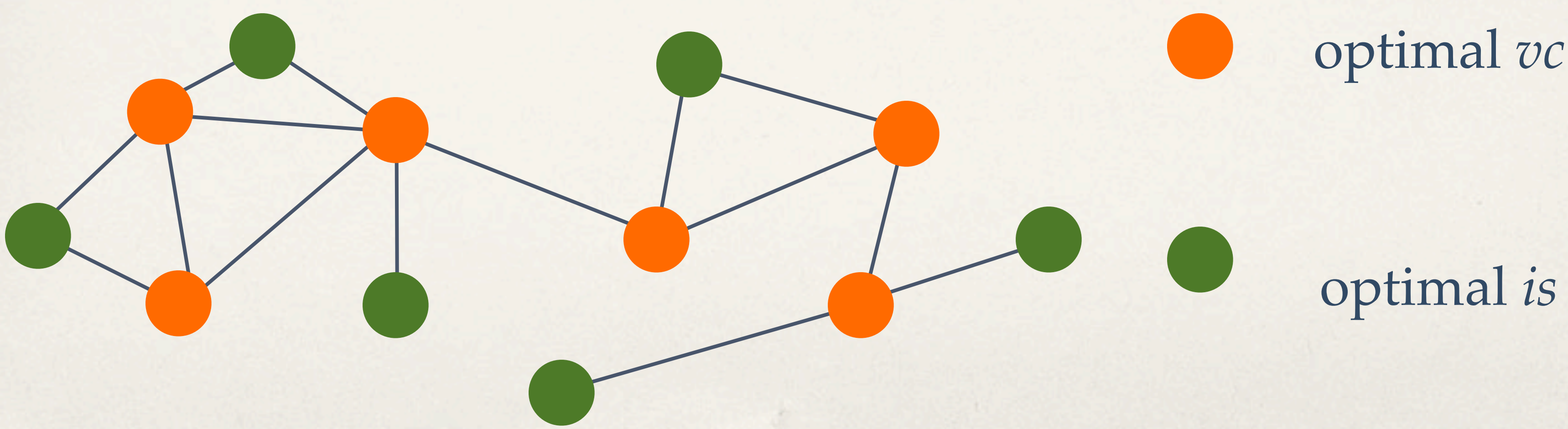


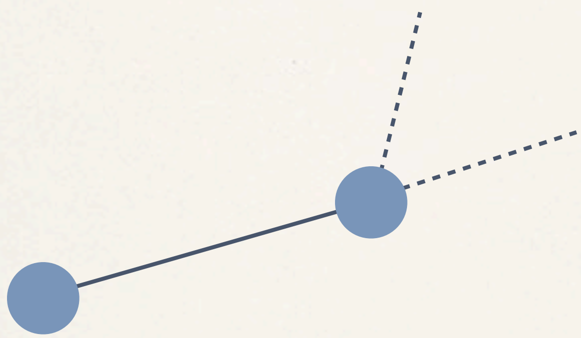
Vertex Cover—Participant Instructions

- Graph as art gallery (rooms = nodes, corridors = edges); optimization version
  - Find the fewest guards necessary to protect the art in the corridors of the art gallery

Independent Set—Instructions

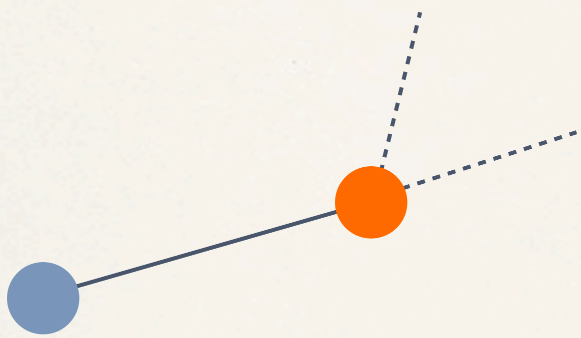
- Social network: Independent people problem; optimization version

Reduction Rules studied



RR1

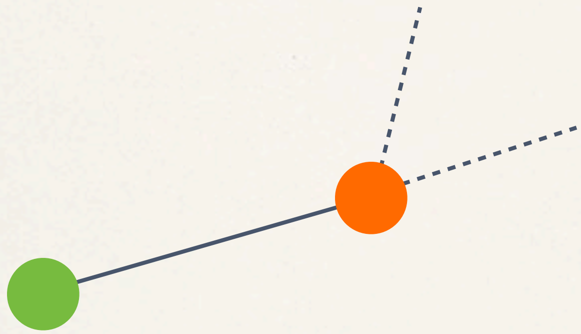
Reduction Rules studied



optimal *vc* selection

RR1

Reduction Rules studied



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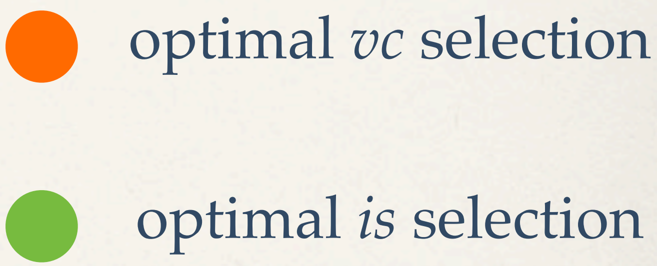
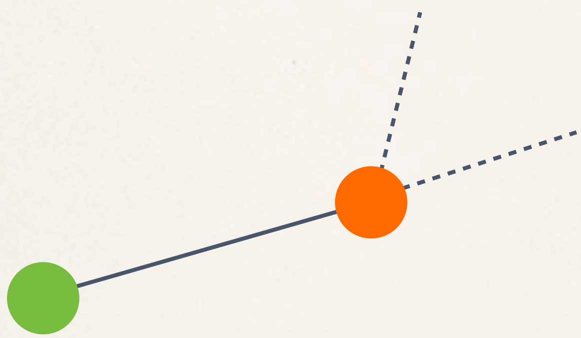


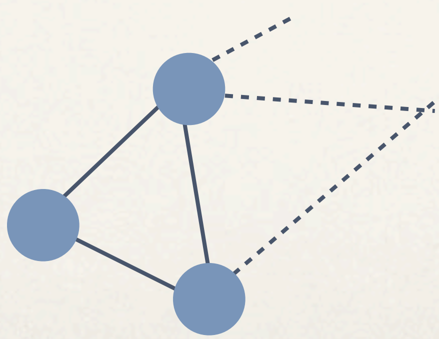
Reduction Rules studied



RR1

optimal *vc* selection

optimal *is* selection

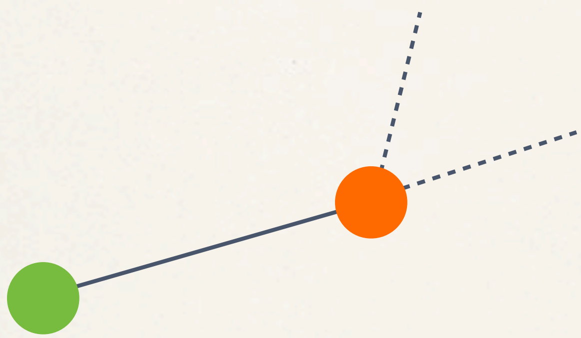


RR2

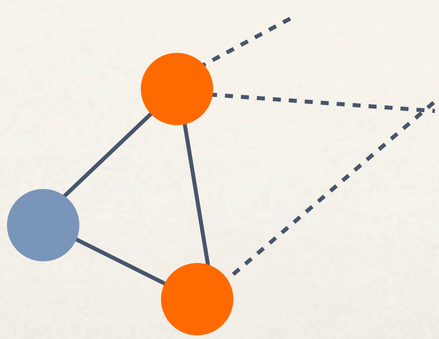
Reduction Rules studied

optimal *vc* selection

optimal *is* selection

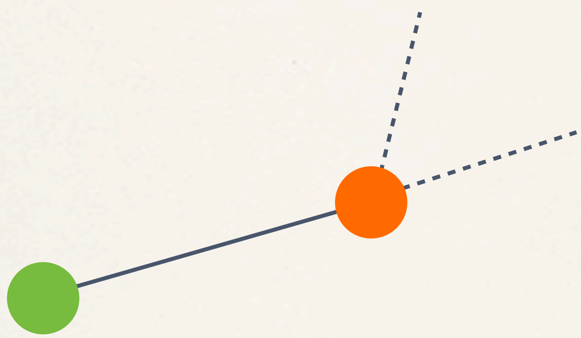


RR1



RR2

Reduction Rules studied



RR1

optimal *vc* selection

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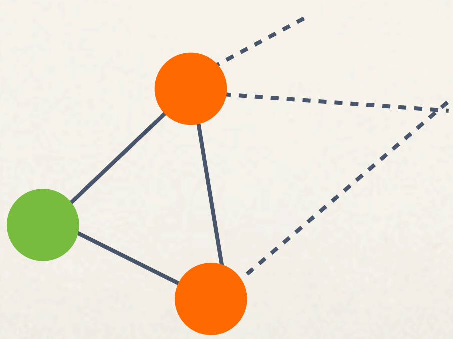


RR2

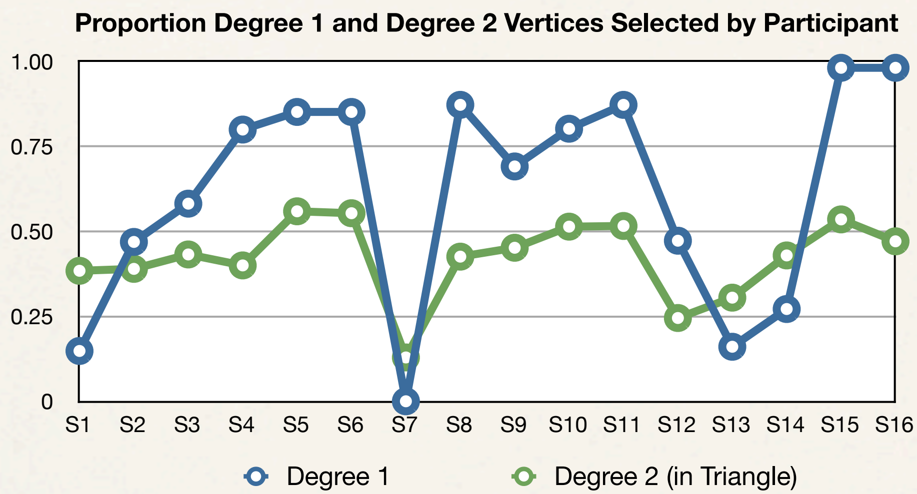
Vertex Cover—Reduction Rule Recognition by Participants

- **Experiment 1: 14 participants (non-CS/MATH), 24 instances [**Carruthers, Masson, S., 2012**]; Vertex Cover only**
- **Experiment 2: 16 participants each (non-CS/MATH), 16 instances; Vertex Cover vs. Independent Set**
  - **RR1**
    - **highly likely** to choose neighbors of pendant vertices to add to vertex cover when available.
    - participants are likely able to identify RR1

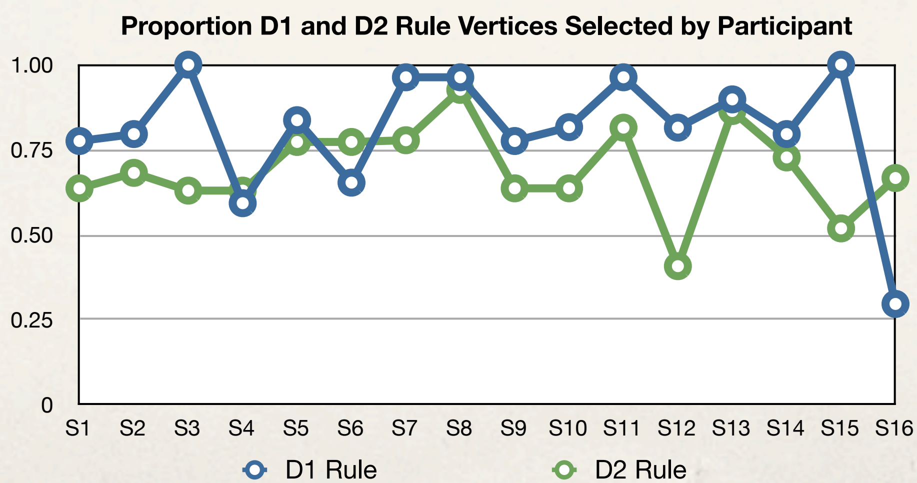
Vertex Cover—Reduction Rule Recognition by Participants

- **Experiment 1: 14 participants (non-CS/MATH), 24 instances [**Carruthers, Masson, S., 2012**]; Vertex Cover only**
- **Experiment 2: 16 participants each (non-CS/MATH), 16 instances; Vertex Cover vs. Independent Set**
  - **RR2**
    - **likely** to choose to add neighbors of degree-2 vertices in triangles where available

Independent Set vs. Vertex Cover: Reduction Rule Selection (Experiment 2)

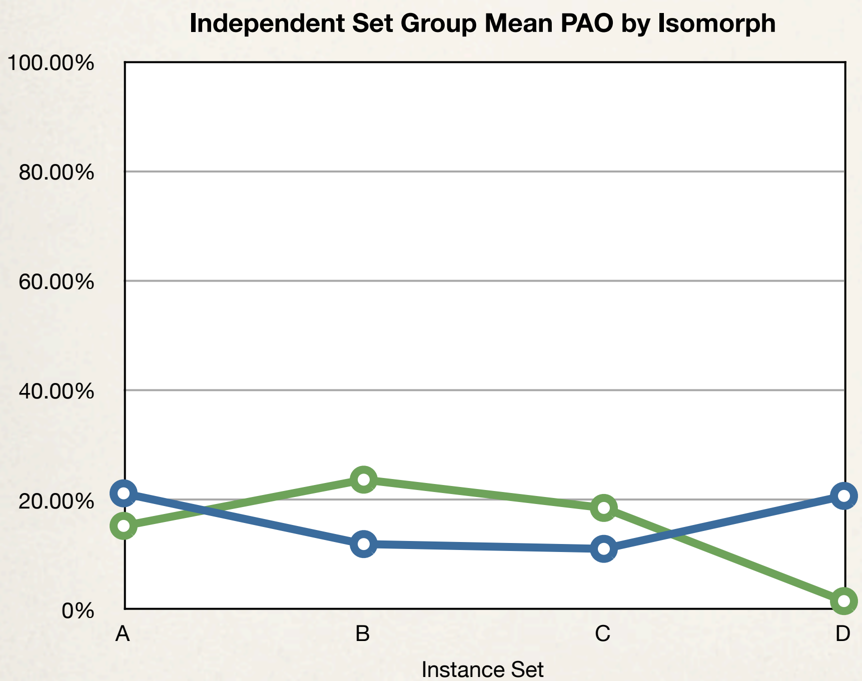


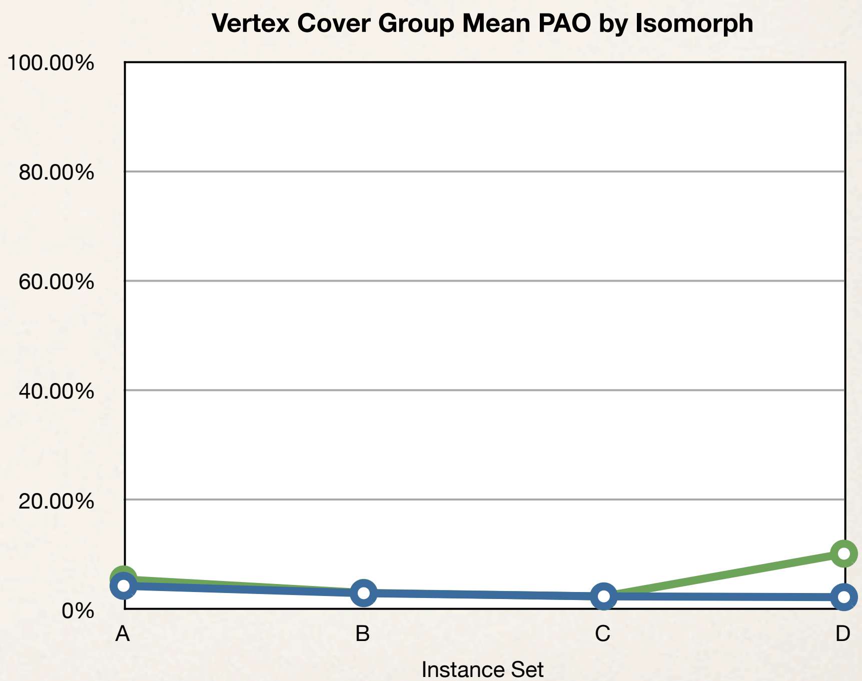
Independent Set



Vertex Cover

Solution quality: IS vs. VC





Summary

- In preliminary study
  - Solving VC appears easier than IS
    - people are better at solving VC than IS
  - RR1 picked up on pretty consistently for both VC and IS
  - RR2 did much better for VC
- Next step: Investigate reasons why IS is solved not as good

Conclusions and Challenges

- Tractability of cognitive functions
  - full array of parameters worth to be studied (suggested by Wareham)
  - which parameters are picked up in human problem solving strategies?
- "Equivalent" problem formulations may impact cognitive tractability
- Tractable Cognitions Thesis: Tractable $\subseteq$ FPT may be too narrow (e.g. TSP)
  - e.g. visual properties (E-TSP)
- Cognitive tractable $\subseteq$ P-size kernelizable + some other properties?