Incremental List Coloring of Graphs, Parameterized by Conservation[2]

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[2] Paper to appear in *Theoretical Computer Science*.

Leitmotif: Evolution Instead of Revolution!

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but **yes** to

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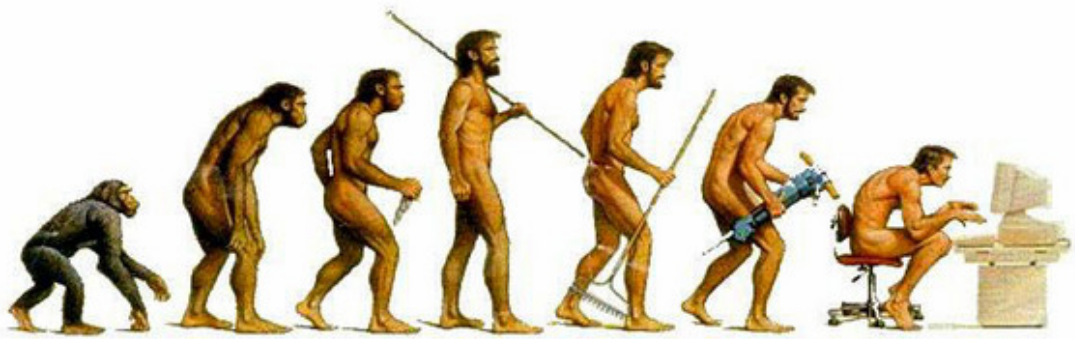

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Basic contents:

- Parameterized local search in terms of "conservative parameterization" for graph (list) coloring.
- Proof of concept for practical relevance through preliminary experiments in a heuristic graph coloring context.

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Basic message:
Desire for **evolution instead of revolution** provides opportunities for research on parameterized and multivariate complexity analysis.

Motivation, Stream 1: (Parameterized) Local Search

A famous example:

Travelling Salesperson Problem (TSP)

Input An edge-weighted graph G.

Task Find a minimum weight Hamiltonian cycle in G.

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Local search variant:

Local TSP

Input An edge-weighted graph $G = (V, E)$, an integer k, and a labeling such that $id = v_1, v_2, \ldots, v_n, v_1$ is a Hamiltonian cycle in G.

Question Is there an improved Hamiltonian cycle within the k-neighborhood of id in G?

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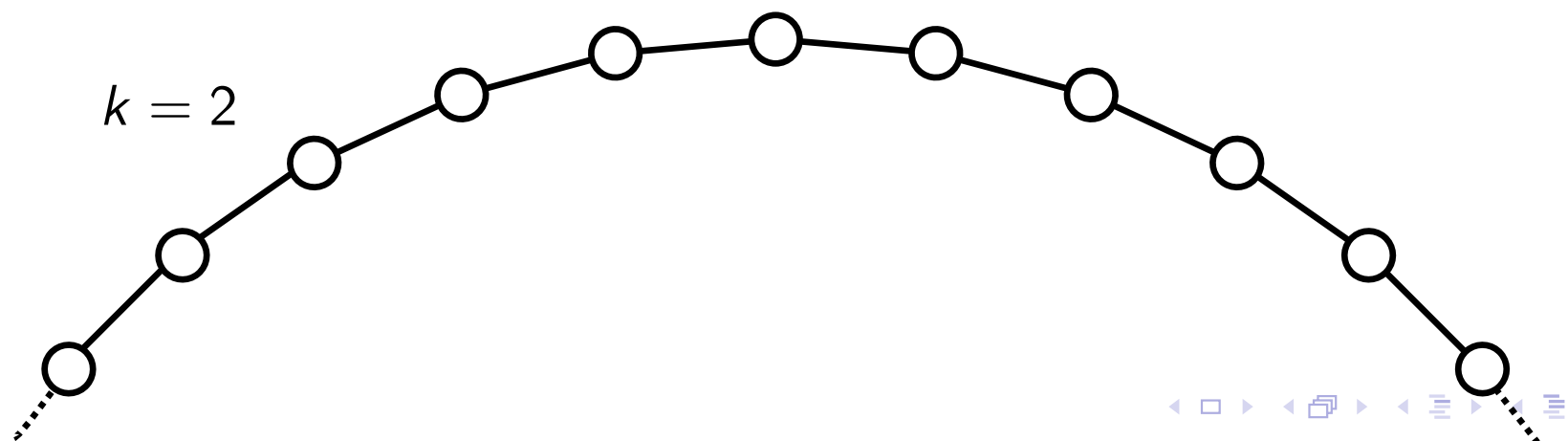
How to define the k-neighborhood of a Hamiltonian cycle?
$\rightsquigarrow$ k-**Edge** neighborhood: exchange at most k edges of id.



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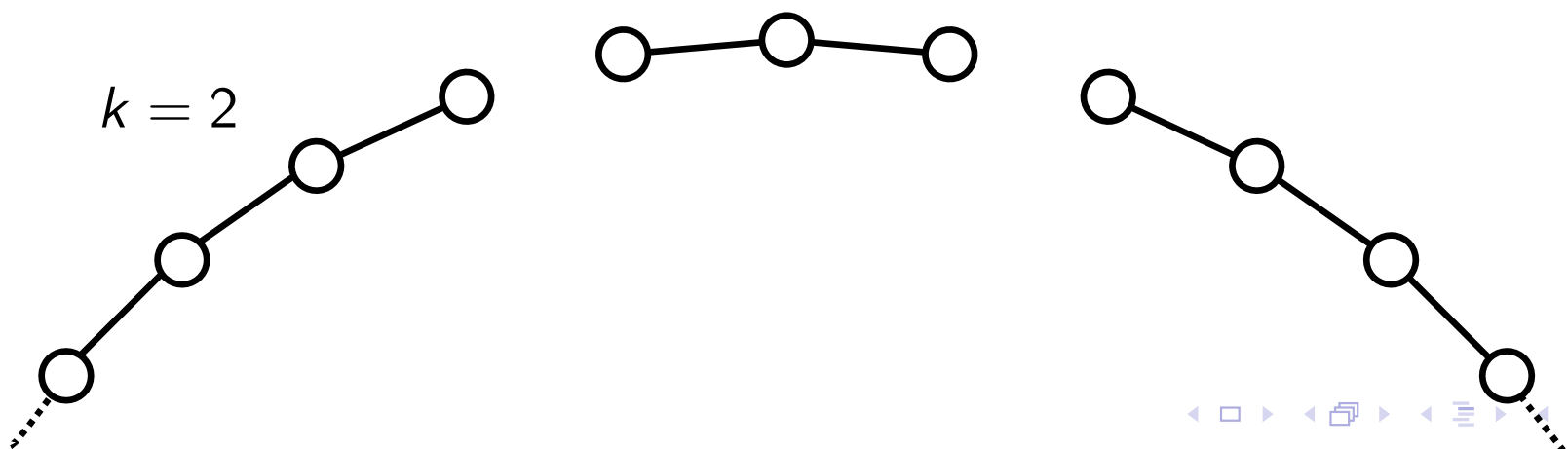

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Running time for Local TSP(Edge)?

- Straightforward brute-force gives $n^{k+O(1)}$ running time.
- Question: $\exists\ f(k) \cdot n^{O(1)}$-time algorithm (FPT)?
  - No! $\rightarrow$ Marx [Oper. Res. Letters 2008]:
    Searching the k-change neighborhood for TSP is W[1]-hard.

Motivation, Stream 1: Local TSP II

Neighborhood measures besides edge neighborhood:

Max-Shift: Move every vertex at most k positions apart.
$\rightsquigarrow$ Balas [Annals of OR 1999]:
$O(4^{k-1} \cdot k^{1.5} \cdot n)$ time algorithm for Local TSP(Max-Shift)

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Further measures of interest:
For $id = 1, 2, \ldots, n$, define the following operations:

- swap(i, j) results in $1, \ldots, i-1, j, i+1, \ldots, j-1, i, j+1, \ldots, n$;
- edit(i, j) results in $1, \ldots, i-1, i+1, \ldots, j-1, j, i, j+1, \ldots, n$;
- reversal(i, j) results in $1, \ldots, i-1, j, j-1, \ldots, i+1, i, j+1, \ldots, n$.

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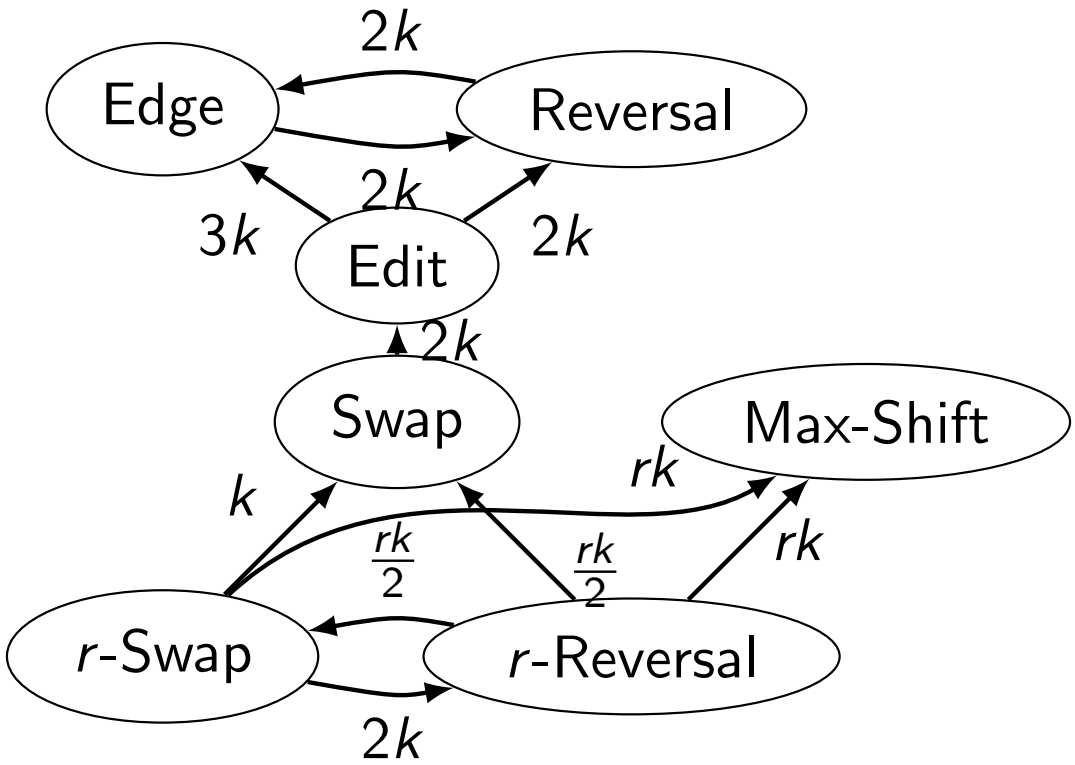
- reversal(i, j) results in $1, \ldots, i-1, j, j-1, \ldots, i+1, i, j+1, \ldots, n$.

Applying at most k operations leads to **Swap**, **Edit**, and **Reversal** k-neighborhood measures.

(Moreover, $0 < j - i < r \rightsquigarrow$ r-**Swap** and r-**Reversal**.)

Motivation, Stream 1: Local TSP III

Parameter space of Local TSP distance measures, partially ordered [Guo, Hartung, N., Suchý; ISAAC 2011, Algorithmica online available]:



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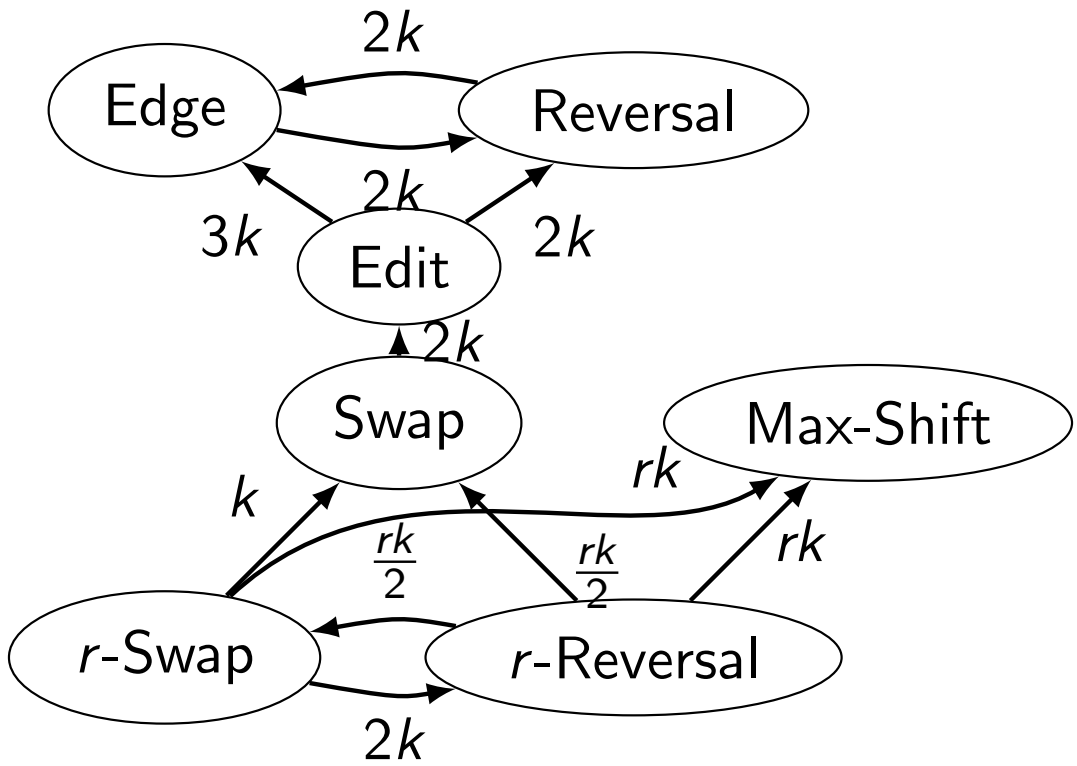

graphs	Swap	r-Swap	Edit	Reversal	r-Reversal	Edge
general	W[1]-h	FPT	W[1]-h	W[1]-h	FPT	W[1]-h
planar	FPT	FPT	FPT	?	FPT	?

Motivation, Stream 2: (Parameterized) Incrementalization

Incremental Clustering and Dynamic Information Retrieval
[Charikar, Chekuri, Feder, Motwani; STOC 1997; SICOMP 2004]:

$\rightsquigarrow$ **Incremental Clustering** problem:
For an update sequence of n points in metric space, maintain a collection of k clusters such that as each input point is presented, either it is assigned to one of the current k clusters or it starts off a new cluster while two existing clusters are merged into one.

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- Practial motivation: Maintain clusters in dynamic environments; updating clusterings without performing frequent reclustering is highly desirable.

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Remarks:

- Practial motivation: Maintain clusters in dynamic environments; updating clusterings without performing frequent reclustering is highly desirable.
- Closely related to *online* clustering model, but incremental model is more restrictive (adversary does not need to respect input order...).
- Charikar et al. focus on polynomial-time approximation and investigate several variants of Incremental Clustering.

Motivation, Stream 2: k-Center and k-List Coloring

k-Center

Input A distance function d on an element set X, $t \in \mathbb{R}^+$ and $k \in \mathbb{N}^+$.

Question Is there a k-partition $C_1, \ldots, C_k$ of X such that $\max_{1 \leq i \leq k} \max_{v,u \in C_i} d(v,u) \leq t$?

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k-List Coloring

Input A graph $G = (V, E)$, $k \in \mathbb{N}^+$ and a list of colors $L(v) \subseteq \{1, \ldots, k\}$ for each $v \in V$.

Question Is there a k-list coloring $f : V \to \{1, 2, \ldots, k\}$ of G?

f is a k-**list coloring** of G iff $\forall u, v \in E : f(u) \neq f(v)$ and $\forall v \in V : f(v) \in L(v)$.

Motiv., Stream 2: Incremental Conservative k-List Coloring

Incremental Conservative k-List Coloring (IC k-List Coloring)

Input A graph $G = (V, E)$, k-list coloring f for $G[V \setminus \{x\}]$, and number $c \in \mathbb{N}$ of allowed recolorings.

Question Is there a k-list coloring f' for G such that $|\{v \in V \setminus \{x\} \mid f(v) \neq f'(v)\}| \leq c$?

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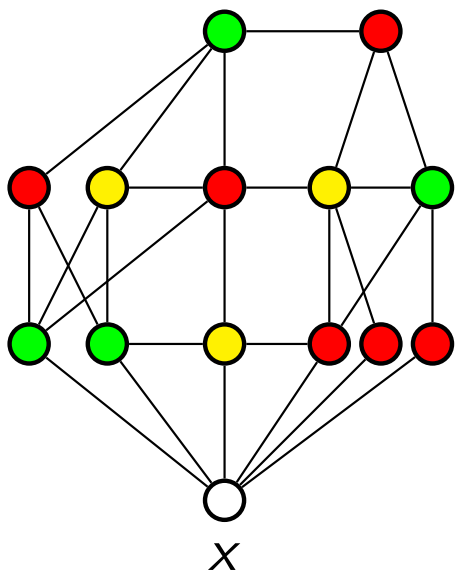

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$L(v) \subseteq \{1, \ldots, k\}\ \forall v \in V$

Note: c measures degree of change allowed: *conservation* parameter.

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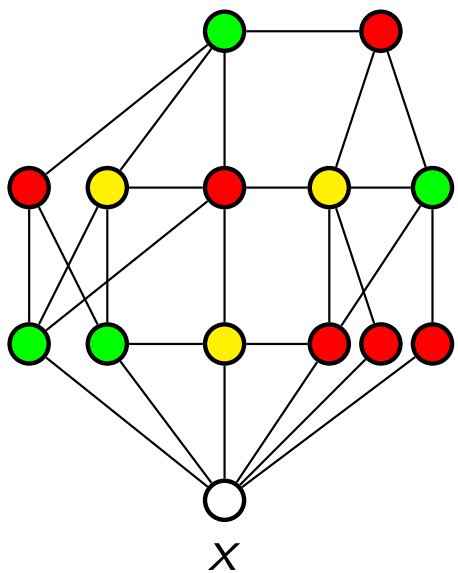

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Parameterize on conservation!

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Theorem. IC 3-Coloring is NP-complete even on bipartite graphs.

Idea of proof. Use corresponding NP-hardness result for "Precoloring Extension" due to Bodlaender, Jansen, and Woeginger [Discrete Appl. Math. 1994]. □

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Idea of kernelization:

- Compute a shortest "**possible conflict path**" P_v for $v \in V$.
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Idea of no-poly-kernel result:
"Standard use" of Bodlaender, Downey, Fellows, and Hermelin's framework [JCSS 2009]. Show "OR-compositionality".

Kernelization for IC k-List Coloring II

Let S be set of $\leq c+1$ recolored vertices, including x.
W.l.o.g. $G[S]$ is connected.

Conflict path for $v_j \in S$: path $[x, v_1, \ldots, v_j]$ such that $f'(x) = f(v_1)$ and $f'(v_i) = f(v_{i+1})\ \forall 1 \leq i < j$

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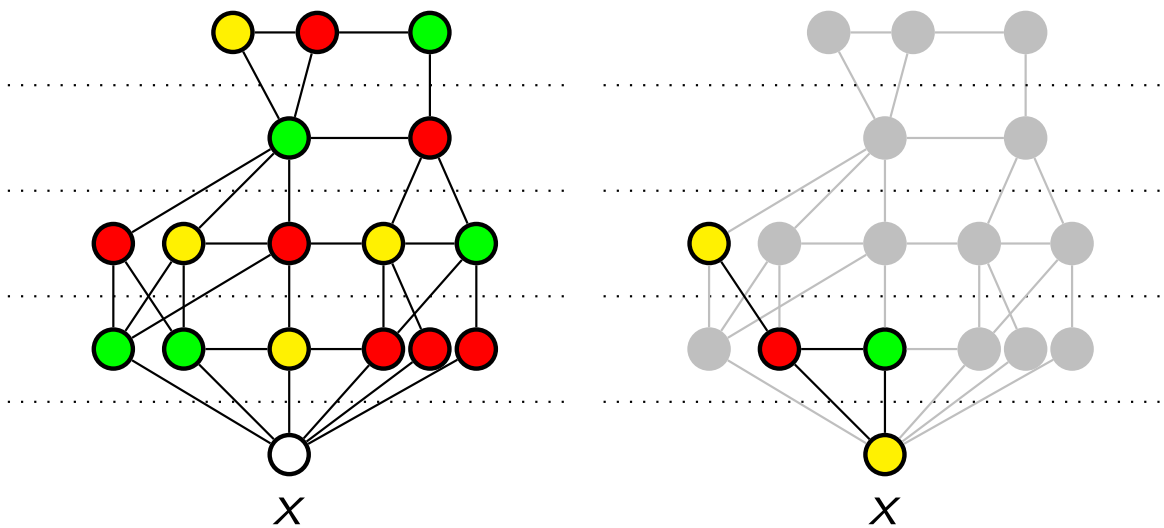

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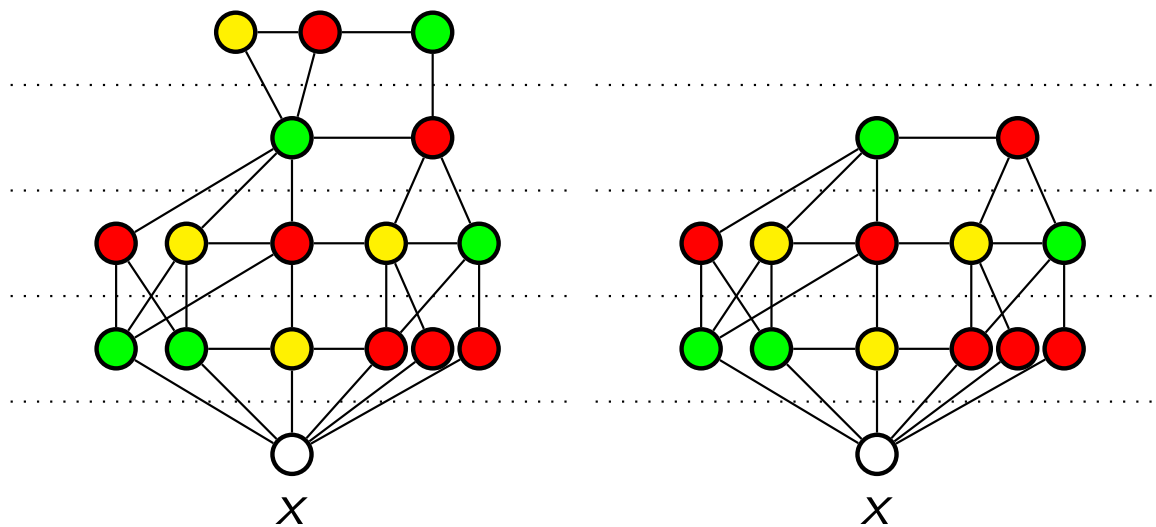

IC k-List Coloring on Special Graphs

class	IC k-List Col.		IC k-Col.	PrExt	k-List Col.
	poly kernel				
trees	/	P	P	P	P
compl. bip.	?	NP*-c	P	**P**	**NP-c**
bipartite	no	NP-c	NP-c	**NP-c**	NP-c
chordal	?	NP*-c	NP-c	NP-c	NP-c
interval	?	NP*-c	?	**NP-c**	NP-c
unit interval	yes	NP*-c	?	**NP-c**	NP-c
cographs	?	?	?	**P**	**NP-c**
dist.-hered.	?	NP*-c	NP-c	**NP-c**	NP-c
split	?	NP*-c	?	**P**	**NP-c**

NP*-c: Turing reductions used; boldfaced results from literature.

IC k-List Coloring on Special Graphs

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trees	/	P	P	P
compl. bip.	?	NP*-c	P	**P**	**NP-c**
bipartite	no	NP-c	NP-c	**NP-c**	NP-c
chordal	?	NP*-c	NP-c	NP-c	NP-c
interval	?	NP*-c	?	**NP-c**	NP-c
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dist.-hered.	?	NP*-c	NP-c	**NP-c**	NP-c
split	?	NP*-c	?	**P**	**NP-c**

NP*-c: Turing reductions used; boldfaced results from literature.

Note: Using dynamic programming, IC k-List Coloring can be solved in $O(k^{\omega+1}\omega^2 \cdot |V|)$ time on graphs of treewidth ω (with given tree dec.).

Proof of Concept: Experimental Work I

Setting: Using the solution to IC k-List Coloring in a subroutine of a well-known **greedy** heuristic for graph coloring yields promising results.

Idea: If greedy (color according to descending vertex degree) fails, then try to conservatively recolor with $c \leq 8$.

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Compared with **Iterated Greedy** heuristic due to Culberson and Luo [DIMACS Series in Discrete Math. and Theor. Comput. Sci., 1996]: Iteratively run the greedy algorithm by trying different vertex orderings in the coloring process...; abort when after 1000 iterations no better coloring was found.

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Tested on 64 benchmark graph instances (between 25 and 4730 vertices and 15 % average edge density), taken from
DIMACS "Graph Coloring and its Generalizations" Symposium 2002.

Proof of Concept: Experimental Work II

Each value obtained as avg. over four runs (std. deviation in brackets):

	greedy		Iterated Greedy			search tree		
	k	time	k	#iter	time	k	c	time
gr1	8	0.2	5.0 [0.0]	1058.8	33.2 [1.3]	5.0 [0.0]	8	0.2 [0.0]
gr2	29	0.1	27.5 [0.6]	1243.8	24.5 [5.0]	25.0 [0.0]	6	0.5 [0.1]
gr3	17	0.0	16.8 [0.5]	1217.5	6.7 [2.3]	14.8 [0.5]	8	0.3 [0.1]
gr4	148	0.3	109 [1.4]	2765.3	68.8 [9.2]	116.8 [2.6]	4	1.5 [0.6]
gr5	18	0.0	18.0 [0.0]	1000	5.0 [0.0]	16.0 [0.0]	7	1.2 [0.2]
gr6	44	0.3	42.0 [0.0]	1033.8	59.9 [1.7]	41.0 [0.0]	5	4.8 [0.3]
gr7	26	0.0	20.3 [0.5]	1295	2.2 [0.7]	19.3 [0.5]	7	0.4 [0.2]
gr8	31	0.0	14.0 [0.0]	1443.8	3.7 [0.3]	23.0 [5.4]	6	0.2 [0.1]
gr9	55	4.0	53.8 [0.5]	1006.3	475.5 [3.4]	50.0 [0.0]	5	3.9 [0.3]

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gr5	18	0.0	18.0 [0.0]	1000	5.0 [0.0]	16.0 [0.0]	7	1.2 [0.2]
gr6	44	0.3	42.0 [0.0]	1033.8	59.9 [1.7]	41.0 [0.0]	5	4.8 [0.3]
gr7	26	0.0	20.3 [0.5]	1295	2.2 [0.7]	19.3 [0.5]	7	0.4 [0.2]
gr8	31	0.0	14.0 [0.0]	1443.8	3.7 [0.3]	23.0 [5.4]	6	0.2 [0.1]
gr9	55	4.0	53.8 [0.5]	1006.3	475.5 [3.4]	50.0 [0.0]	5	3.9 [0.3]

Findings:

Our conservative search tree algorithm improves greedy result in 89 % of the tested instances, Iterated Greedy in 83 %. Improvement by 12 % resp. 11 % of number of colors used.

Proof of Concept: Experimental Work II

Each value obtained as avg. over four runs (std. deviation in brackets):

	greedy		Iterated Greedy			search tree		
	k	time	k	#iter	time	k	c	time
gr1	8	0.2	5.0 [0.0]	1058.8	33.2 [1.3]	5.0 [0.0]	8	0.2 [0.0]
gr2	29	0.1	27.5 [0.6]	1243.8	24.5 [5.0]	25.0 [0.0]	6	0.5 [0.1]
gr3	17	0.0	16.8 [0.5]	1217.5	6.7 [2.3]	14.8 [0.5]	8	0.3 [0.1]
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Search tree algorithm by a factor of 50 slower than greedy algorithm, Iterative Greedy by a factor of 170.

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- Study more general settings, e.g. with more than one vertex added, vertex deletions, etc.

Evolution is Conservative—Changes Little!