Overlapping AllDifferent Constraint

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FPT for Multiple AllDifferent Constraint

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Background
Constraint programming
Constraint programming
Constraint programming

\[ X_1 \rightarrow A(lice) \]

\[ X_2 \rightarrow B(ob) \]

\[ X_3 \rightarrow C(laudia) \]
Variables/Domains

\[ X_1, \ D(X_1) = \{ A, C \} \]
\[ X_2, \ D(X_2) = \{ A, C \} \]
\[ X_3, \ D(X_3) = \{ B, C \} \]
Constraint programming

$X_1 \rightarrow A(lice)$

$X_2 \rightarrow B(ob)$

$X_3 \rightarrow C(laudia)$
Constraints

\[ X_1 \neq X_2, \; X_1 \neq X_3, \; X_2 \neq X_3 \]
Global Constraints

\textbf{AllDifferent}(X_1, X_2, X_3)
Constraint programming

Big picture
Constraint programming

Problem
Constraint programming

Problem

Variables, Domains, Constraints
Constraint programming

Problem

Variables, Domains, Constraints

Constraint Solver
Constraint programming

Constraint Solver

Variables, Domains, Constraints

Backtracking search
Constraint programming

Backtracking search
Solver reasons about one constraint at a time
Constraint programming

Solver reasons about one constraint at a time
detects inconsistent variable-value pairs for each C
Constraint programming

Solver reasons about one constraint at a time
detects inconsistent variable-value pairs

Propagator
Constraint Propagators
Propagator for a constraint is an algorithm that identifies all its inconsistent variable-value pairs.
Propagators

Strength of a propagator or consistency levels
Consistency levels

Propagator for \( C(X_1, \ldots, X_n) \) can guarantee that

- there exists a \( \text{sol}(C) \)

Detects disentailment
Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- each remaining $X_i = v_j$ can be extended to a $sol(C)$

Domain consistency
Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- exists a $\text{sol}(C)$ over $[\min(D(X_j)), \max(D(X_j))]$

Detections bound disentailment
Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- exists a $sol(C)$ over $[\min(D(X_j)), \max(D(X_j))]$
Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- exists a $sol(C)$ over $[\min(D(X_j)), \max(D(X_j))]$
Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- exists a $\text{sol}(C)$ over $[\min(D(X_j)), \max(D(X_j))]$

```
X_1  X_2  X_3
A(lice)  B(ob)  C(laudia)
X_1 = \{A\}  X_2 = \{B\}  X_3 = \{C\}
```
Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- each remaining $\text{bound}(D(X_i))$ can be extended to a $\text{sol}(C)$ over $[\min(D(X_j)), \max(D(X_j))]$

Bound consistency
Bipartite matching problem

AllDifferent
Constraint programming

\[ X_1 \text{ AllDifferent } X_2 \text{ AllDifferent } X_3 \]

- \( X_1 \)
- \( X_2 \)
- \( X_3 \)

- \( A(lice) \)
- \( B(ob) \)
- \( C(laudia) \)
Constraint programming

AllDifferent

$X_1$

$X_2$

$X_3$

$A(lice)$

$B(ob)$

$C(laudia)$
Bipartite matching problem

Perfect Matching
Bipartite matching problem
Bipartite matching problem
Bipartite matching problem

Let $G = \langle A \cup B, E \rangle$ such that $A \cap B = \emptyset$. There exists a perfect matching iff $|N(P)| \geq |P|$ for $P \subseteq A$. 
Bipartite matching problem

Hopcroft-Karp algorithm runs in $O(E\sqrt{V})$. 
Simultaneous bipartite matching problem

Overlapping AllDifferent
Constraint programming

\[ \begin{align*}
X_1 & \rightarrow X_2 \\
X_2 & \rightarrow X_3 \\
X_3 & \rightarrow X_1 \\
X_1 & \rightarrow X_3 \\
X_3 & \rightarrow X_2 \\
X_2 & \rightarrow X_1
\end{align*} \]
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem

\[ A \quad \{ S \quad T \quad B \} \]
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem.

Simultaneous bipartite matching problem.
Let $G = \langle A \cup B, E \rangle$ and $S$, $T$ be an overlapping bipartite graph. A simultaneous bipartite matching is a set of edges $M \subseteq E$ such that $M \cap (S \times B)$ and $M \cap (T \times B)$ are matchings that cover $S$ and $T$, respectively.
Simultaneous bipartite matching problem

$SBM$ is NP-complete [2005].
Simultaneous bipartite matching problem

Let $G = \langle A \cup B, E \rangle$ and sets $S, T$ be an overlapping bipartite graph. There exists a $SBM$, iff

$$|N(P)| + |N(P_S \setminus P_T) \cap N(P_T \setminus P_S)| \geq |P| \text{ for } P \subseteq A.$$
Simultaneous bipartite matching problem

The problem is NP-complete. Why are we doing all this work?
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem

We solved by reformulation into a system of arithmetic constraints
Simultaneous bipartite matching problem

If the system of arithmetic constraints reaches a non-empty fixpoint then the simultaneous Hall condition holds for $\forall P \subseteq A$. 
Simultaneous bipartite matching problem

SBM on convex graphs can be solved in $O(V^3)$ time.
Simultaneous bipartite matching problem

a faster algorithm for convex graphs.
Simultaneous bipartite matching problem

FPT algorithm for general bipartite graphs.
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem

\[
\forall R \subseteq (S \setminus T) \cup (T \setminus S)
\]
Simultaneous bipartite matching problem

\[ A \] 
\[ S \] 
\[ T \] 
\[ B \]
Simultaneous bipartite matching problem

\[
\begin{align*}
A & \quad \quad S \\
T & \quad \quad B
\end{align*}
\]
Simultaneous bipartite matching problem
Simultaneous bipartite matching problem

\[ A \quad \{ S \} \quad \{ T \} \quad B \]

Check the size for the max matching
Simultaneous bipartite matching problem

FPT algorithm for general bipartite graphs.

\[ O(2^{|S\setminus T| + |T\setminus S|} |S \cup T| |E|) \]
Simultaneous bipartite matching problem

Work in progress:

FPT algorithm for general bipartite graphs.

\[ O(2^{\min(|S \setminus T|, |T \setminus S|)} |S \cup T||E|) \]
We can build a propagator!

FPT algorithm for general bipartite graphs.

\[ O(2^{\min(|S \setminus T|, |T \setminus S|)} |S \cup T||E|) \]
FPT for Multiple AllDifferent Constraint

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Constraint graph
Constraint graph
Constraint graph

![Constraint Graph Image]
Constraint graph

When does CG have a special structure?
Constraint graph

Bad news
Constraint graph

Same-Relation Constraint [CP2009]
Constraint graph

1. Clique
2. Bipartite graph
3. Grid

Same-Relation Constraint [CP2009]
Constraint graph

Good news
Constraint graph

1. CG is induced by MultipleAllDifferent
2. $R$ is $\neq$
3. assumptions on variables domains
4. assumptions on constraints overlaps
Constraint graph

Constraint graph

AllDifferent

X1 X2 X3

AllDifferent

X4 X5 X6

AllDifferent

X7 X8 X9
Constraint graph
1. consecutive cons

Constraint graph
1. consecutive cons

NP-complete for 2 cons
1. consecutive cons

2. interval variables domains
Constraint graph

1. consecutive cons

2. interval variables domains

P for 2 cons
Constraint graph

1. consecutive cons

2. interval variables domains

NP-complete ($\alpha$, $\beta$-list coloring on interval graphs)
1. consecutive cons
2. interval variables domains
3. bounded arity of cons
Constraint graph

1. consecutive cons
2. interval variables domains
3. bounded arity of cons
Constraint graph

1. consecutive cons
2. interval variables domains
3. bounded arity of cons

FPT: the parameter is the max constraint arity
1. consecutive cons
2. interval variables domains
3. bounded arity of cons

FPT: the parameter is the max constraint arity
Constraint graph

1. consecutive cons
2. interval variables domains
3. bounded max domain size

FPT
Thank you!