Overlapping AllDifferent Constraint

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Department of Communications, Information Technology and the Arts
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QUT Queensland University of Technology
THE UNIVERSITY OF QUEENSLAND AUSTRALIA
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FPT for Multiple AllDifferent Constraint

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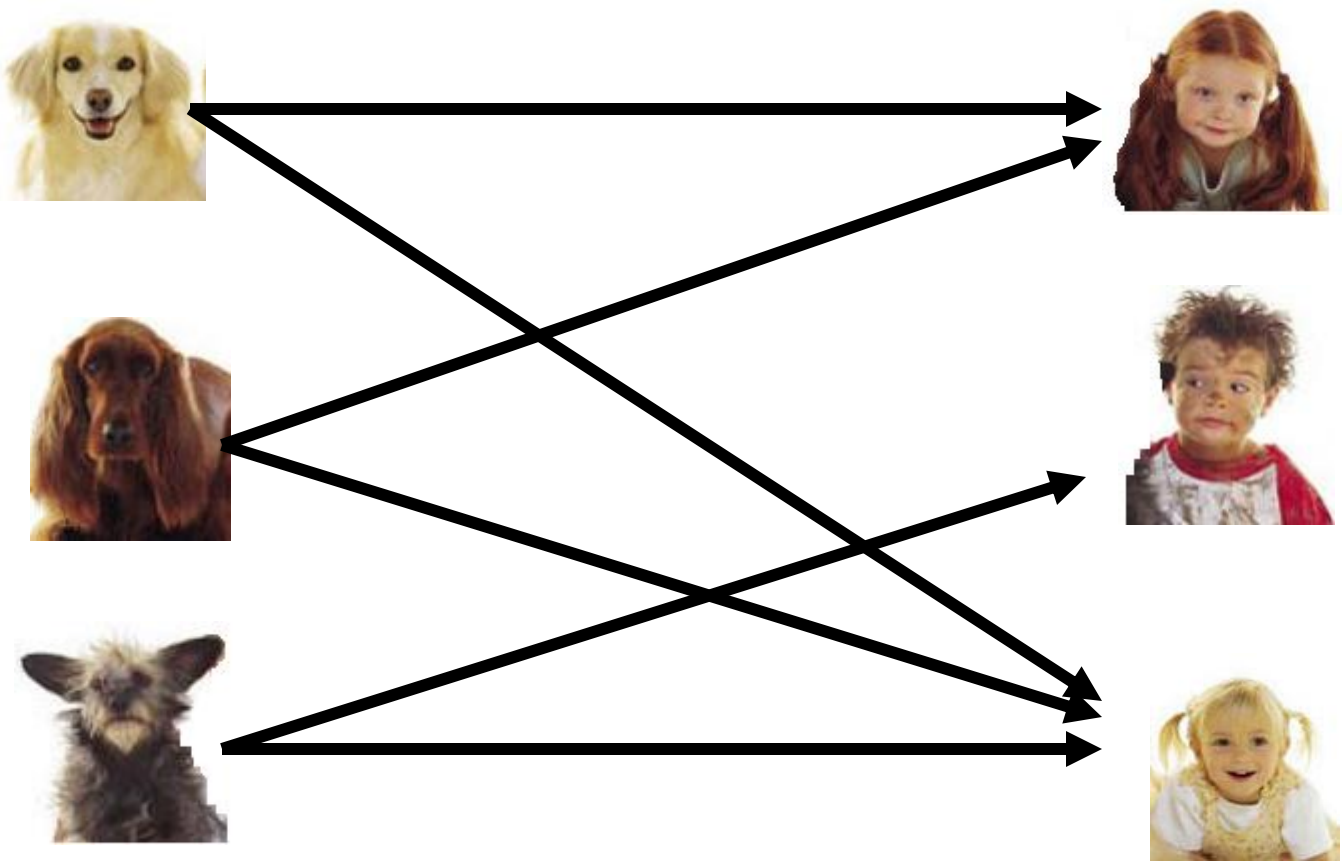
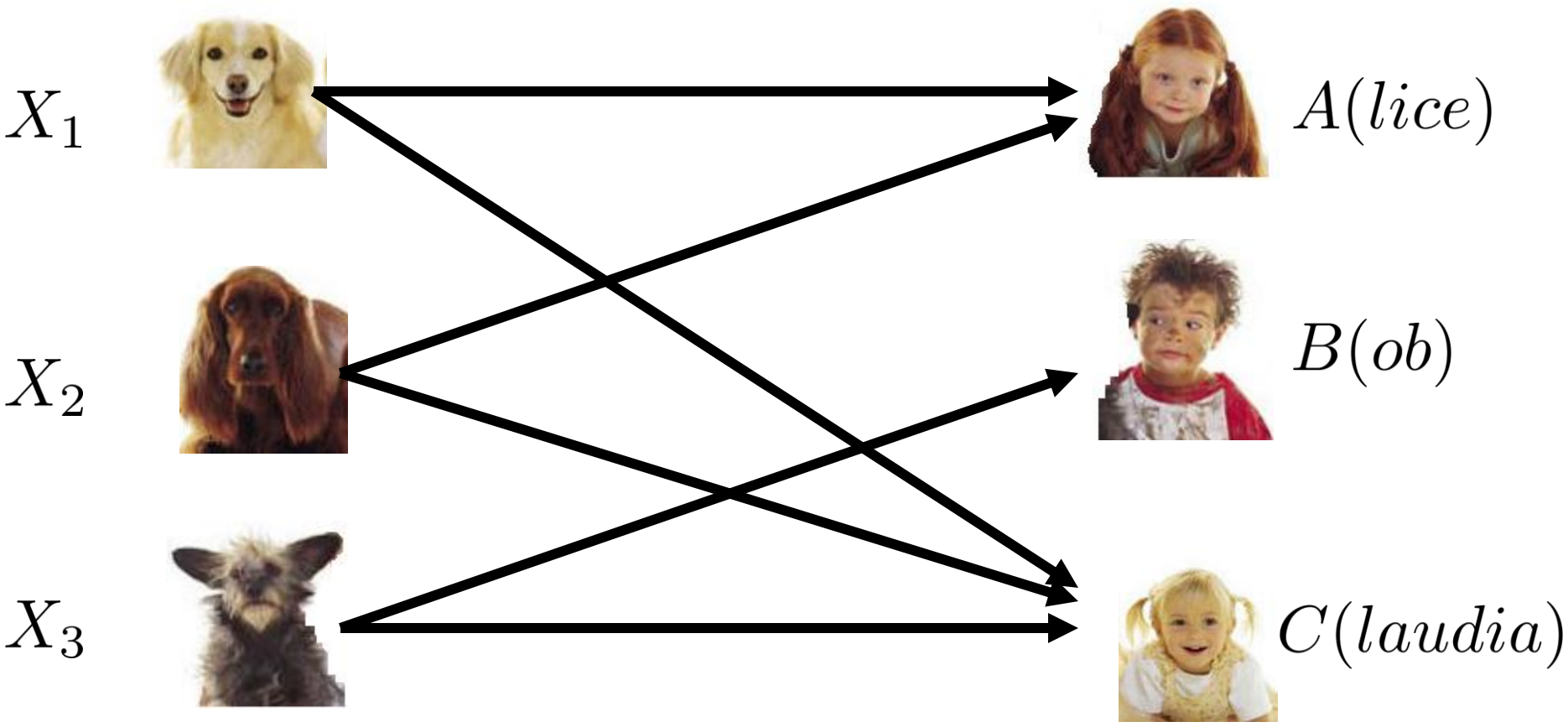
Background

Constraint programming

Constraint programming



Constraint programming

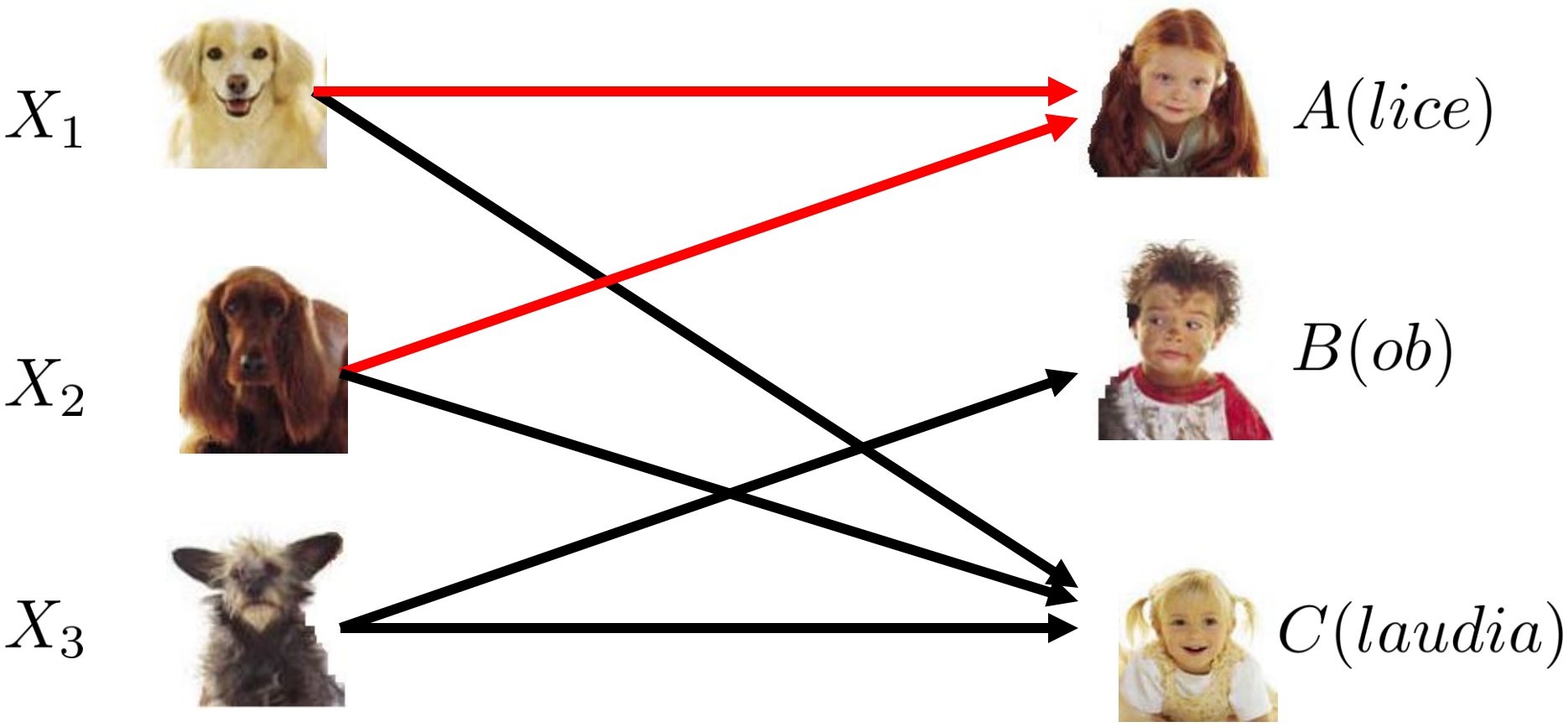


Constraint programming

Variables/Domains

$$X_1,\ D(X_1) = \{A, C\}$$
$$X_2,\ D(X_2) = \{A, C\}$$
$$X_3,\ D(X_3) = \{B, C\}$$

Constraint programming



Constraints

$$X_1 \neq X_2, X_1 \neq X_3, X_2 \neq X_3$$

Global Constraints

AllDifferent(X_1, X_2, X_3)

Constraint programming

Big picture

Constraint programming

Problem

Constraint programming

Problem

Variables, Domains, Constraints

Constraint programming

Problem

Variables, Domains, Constraints

Constraint Solver

Constraint programming

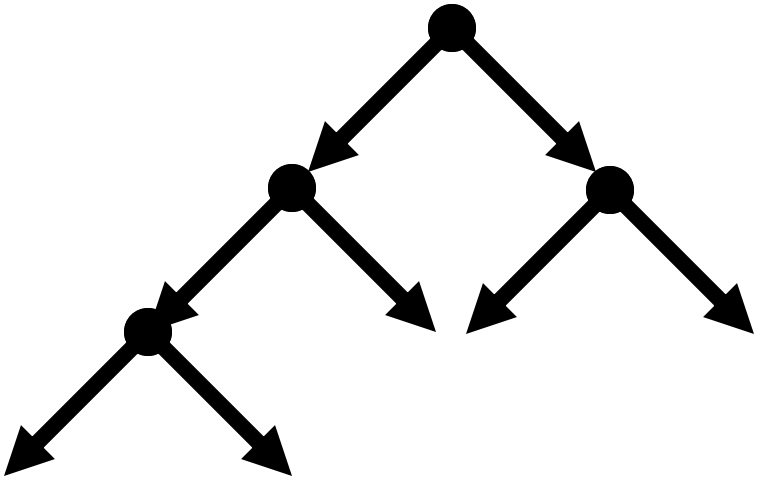
Constraint Solver

Variables, Domains, Constraints

Backtracking search

Constraint programming

Backtracking search



Constraint programming

Solver reasons about one constraint at a time

Constraint programming

Solver reasons about one constraint at a time

detects inconsistent variable-value pairs for each C

Constraint programming

Solver reasons about one constraint at a time

detects inconsistent variable-value pairs from C

Propagator

Constraint Propagators

Propagators

Propagator for a constraint is an algorithm

that identifies all its inconsistent variable-value pairs.

Propagators

Strength of a propagator or consistency levels

Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- there exists a $sol(C)$

Detects disentailment

Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- each remaining $X_i = v_j$ can be extended to a $sol(C)$

Domain consistency

Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

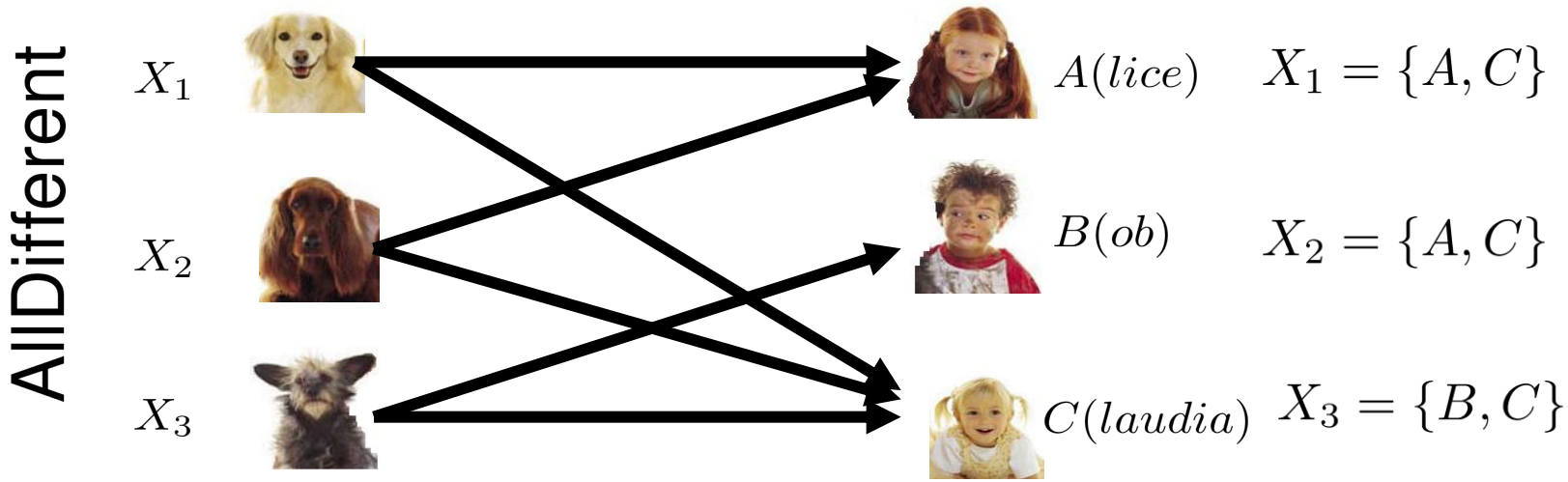
- exists a $sol(C)$ over $[min(D(X_j)), max(D(X_j))]$

  Detects bound disentailment

Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

- exists a $sol(C)$ over $[min(D(X_j)), max(D(X_j))]$



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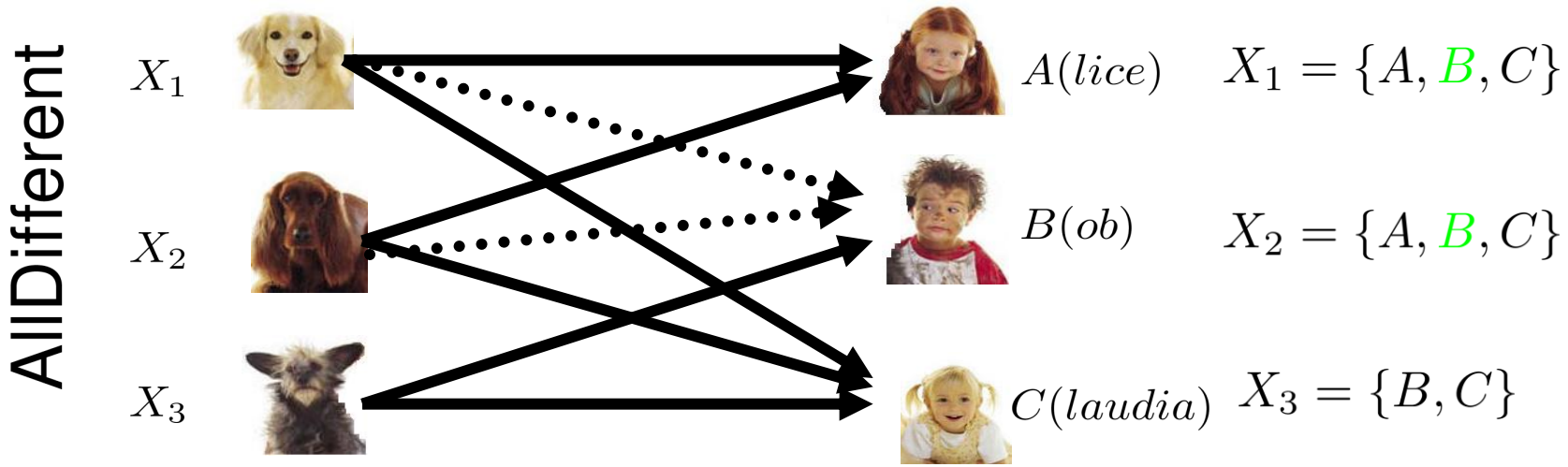

Consistency levels

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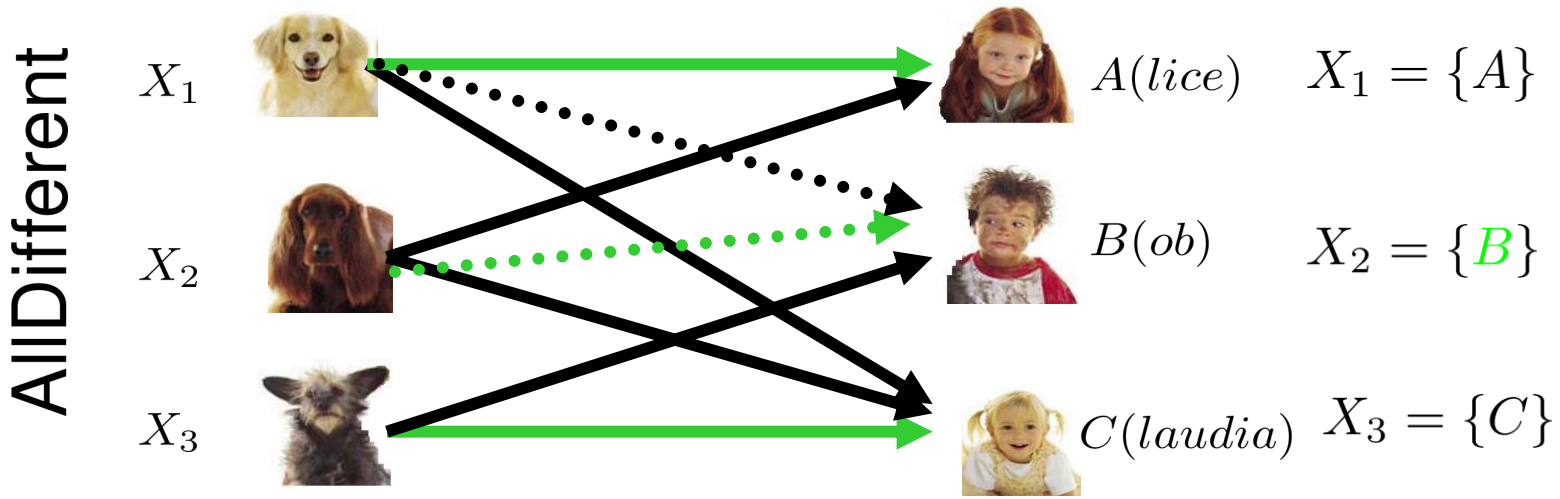


Consistency levels

Propagator for $C(X_1, \ldots, X_n)$ can guarantee that

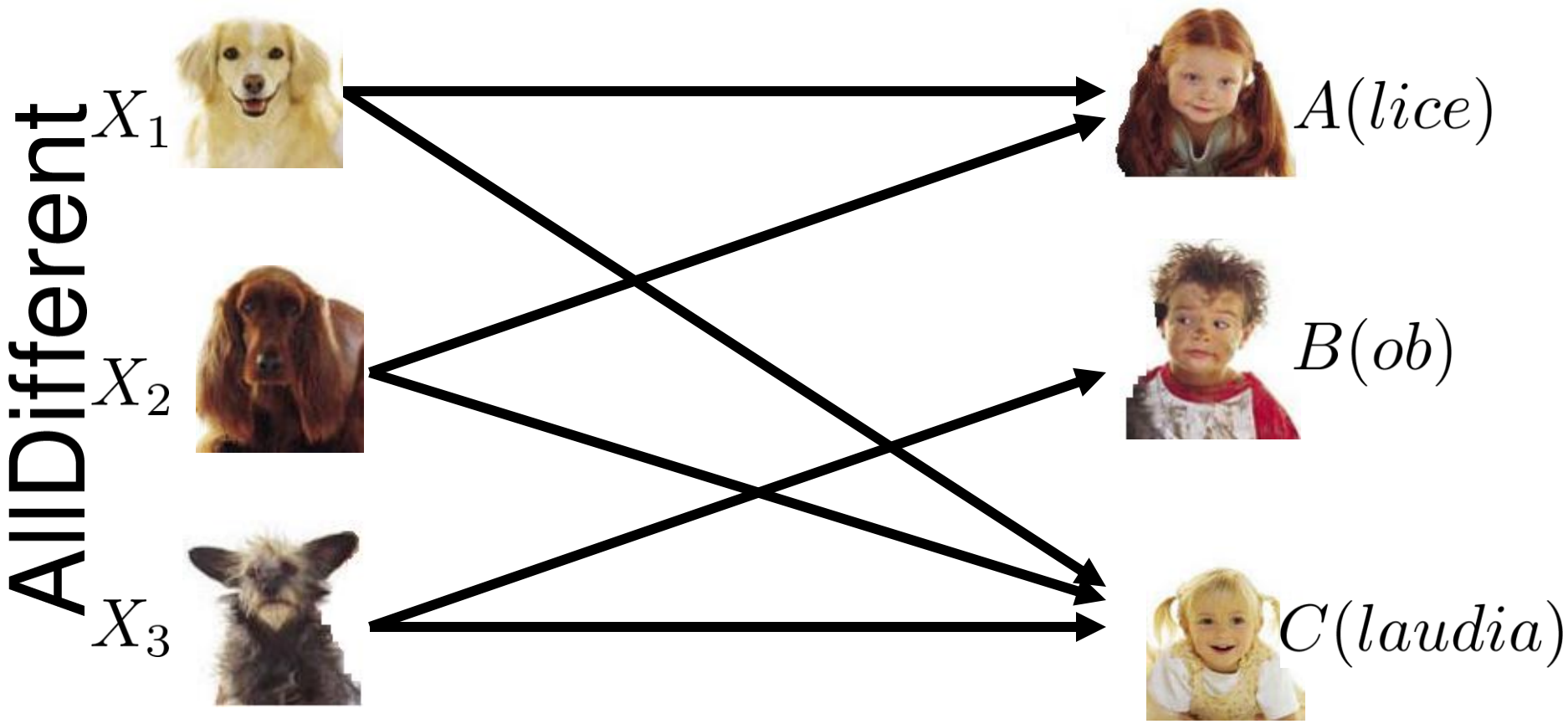
- each remaining $bound(D(X_i))$ can be extended to a $sol(C)$ over $[min(D(X_j)), max(D(X_j))]$

Bound consistency

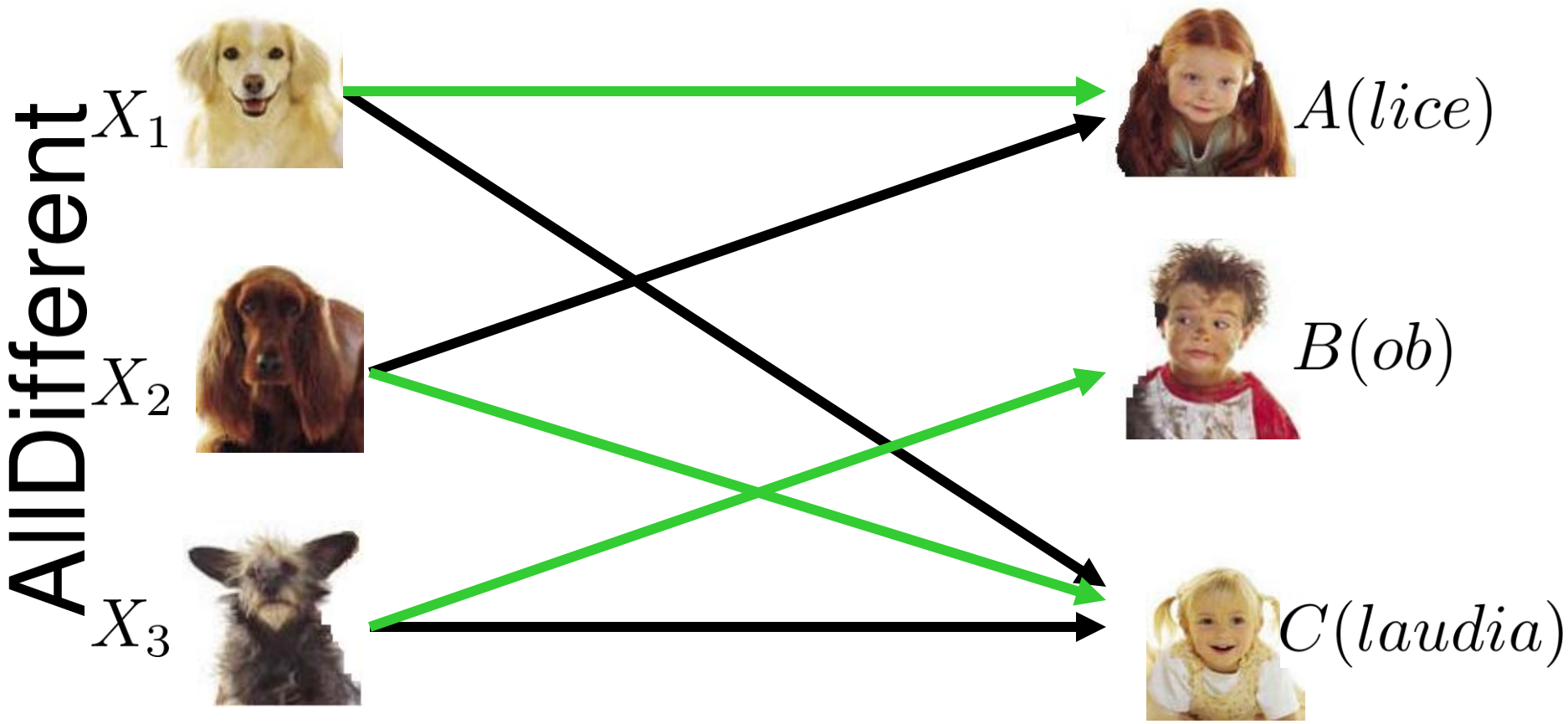
Bipartite matching problem

AllDifferent

Constraint programming



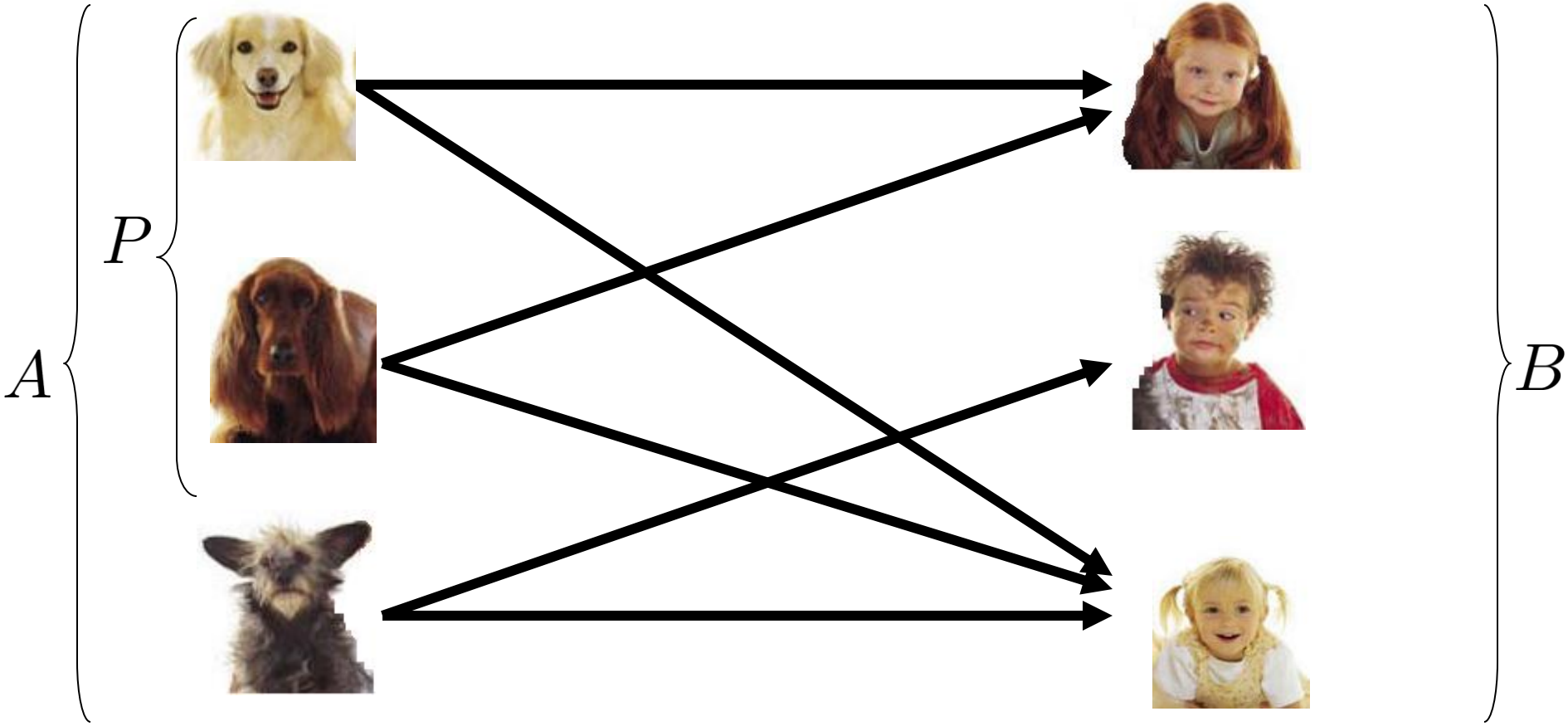
Constraint programming



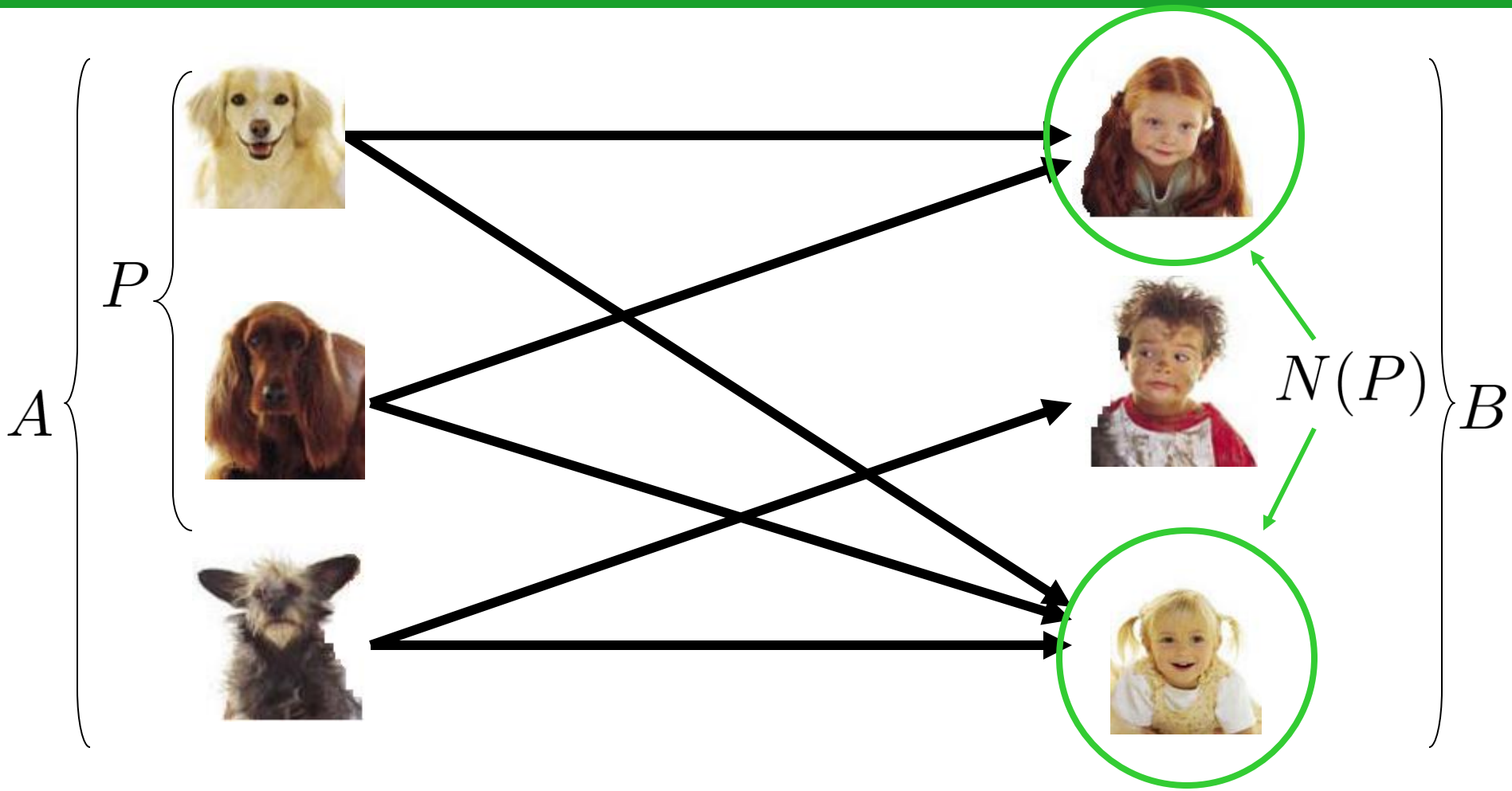
Bipartite matching problem

Perfect Matching

Bipartite matching problem



Bipartite matching problem



Bipartite matching problem

Let $G = \langle A \cup B, E \rangle$ such that $A \cap B = \emptyset$.
There exists a perfect matching iff
$|N(P)| \geq |P|$ for $P \subseteq A$.

Bipartite matching problem

Hopcroft-Karp algorithm runs in $O(E\sqrt{V})$.

Simultaneous bipartite matching problem

Overlapping AllDifferent

Constraint programming

AllDifferent

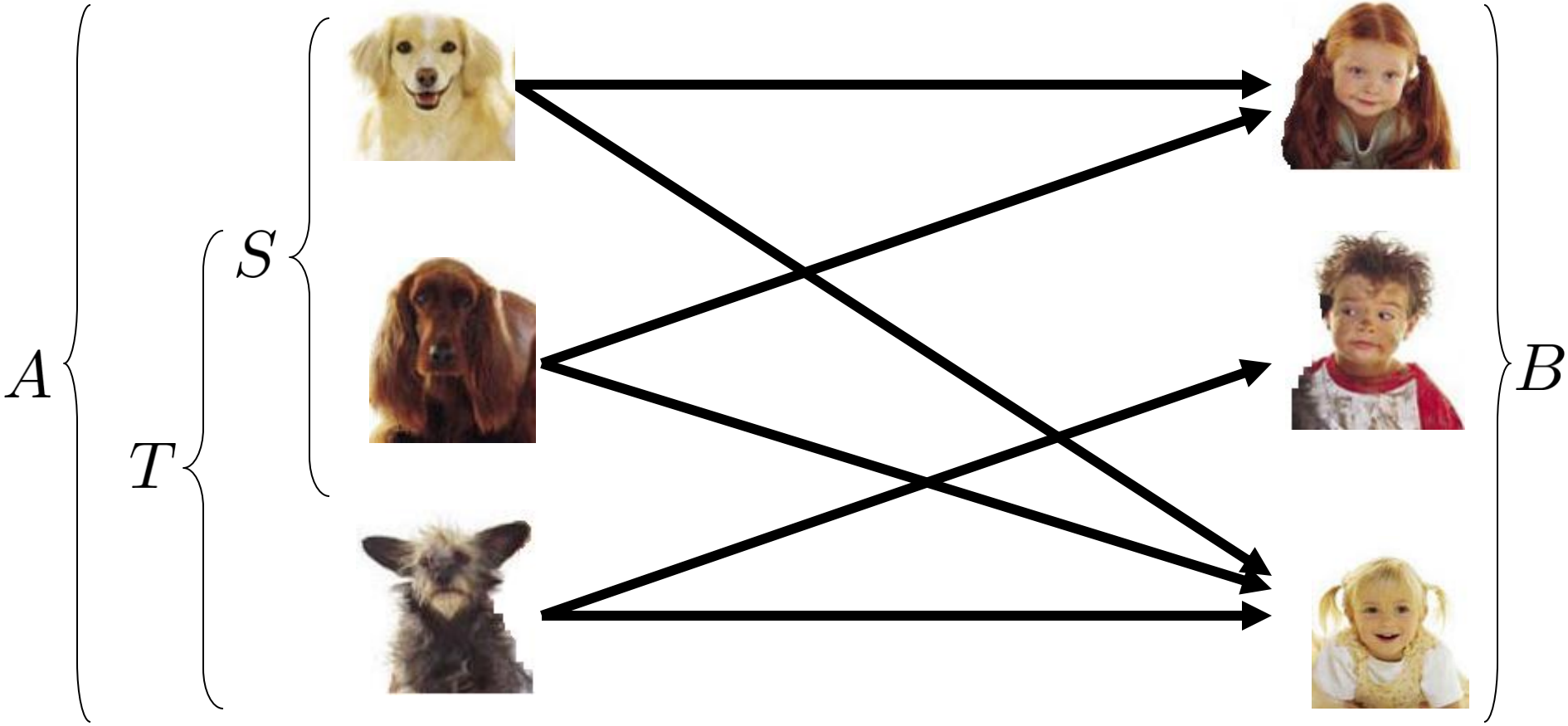
AllDifferent

X_1

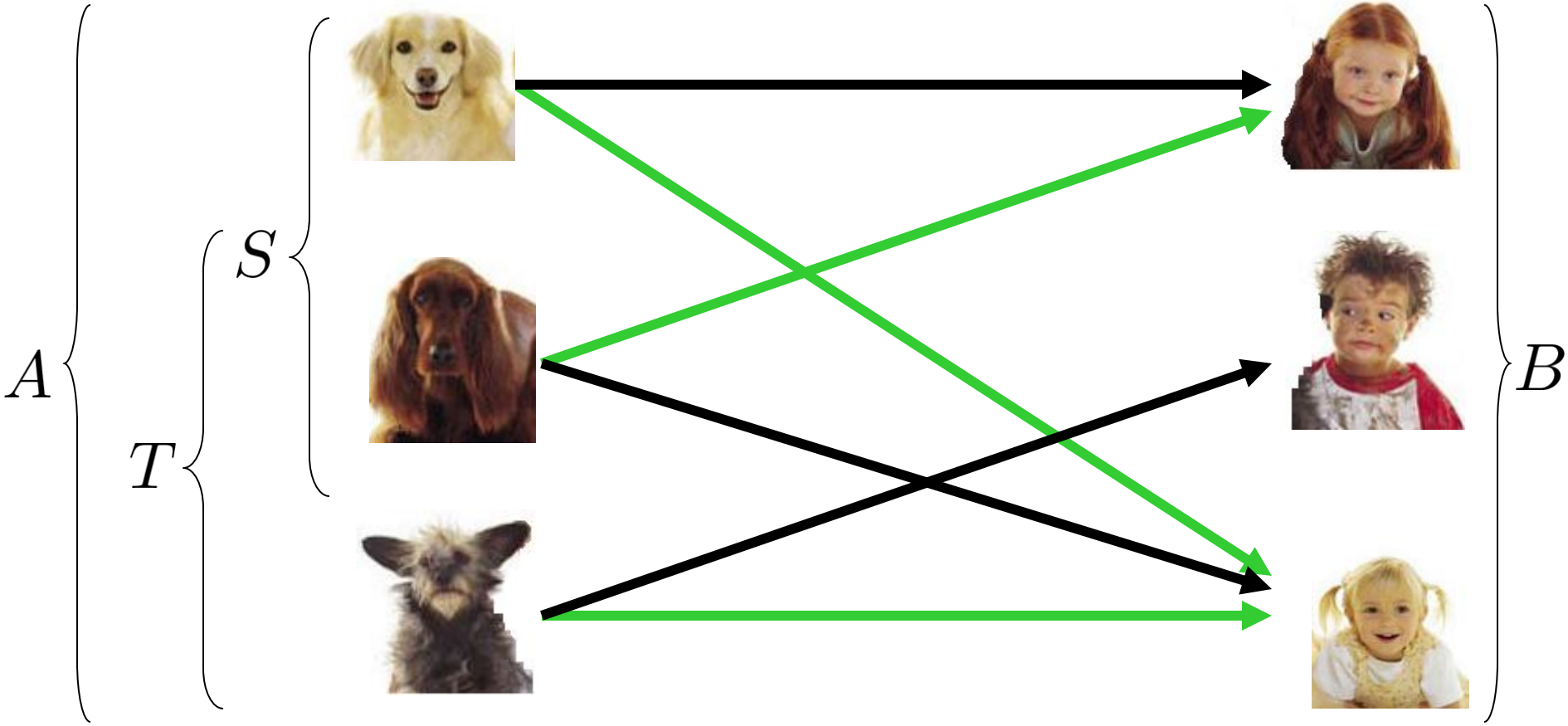
X_2

X_3

Simultaneous bipartite matching problem



Simultaneous bipartite matching problem



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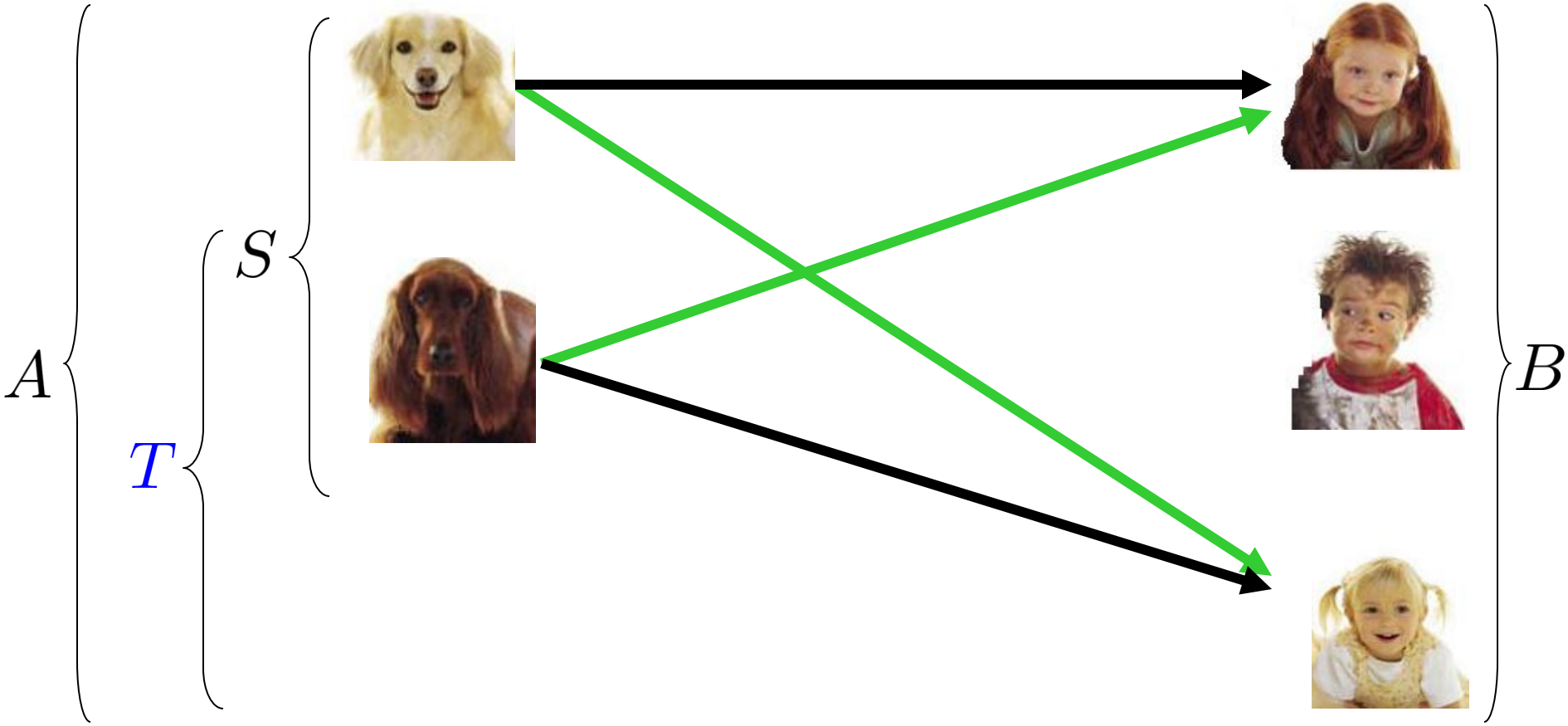

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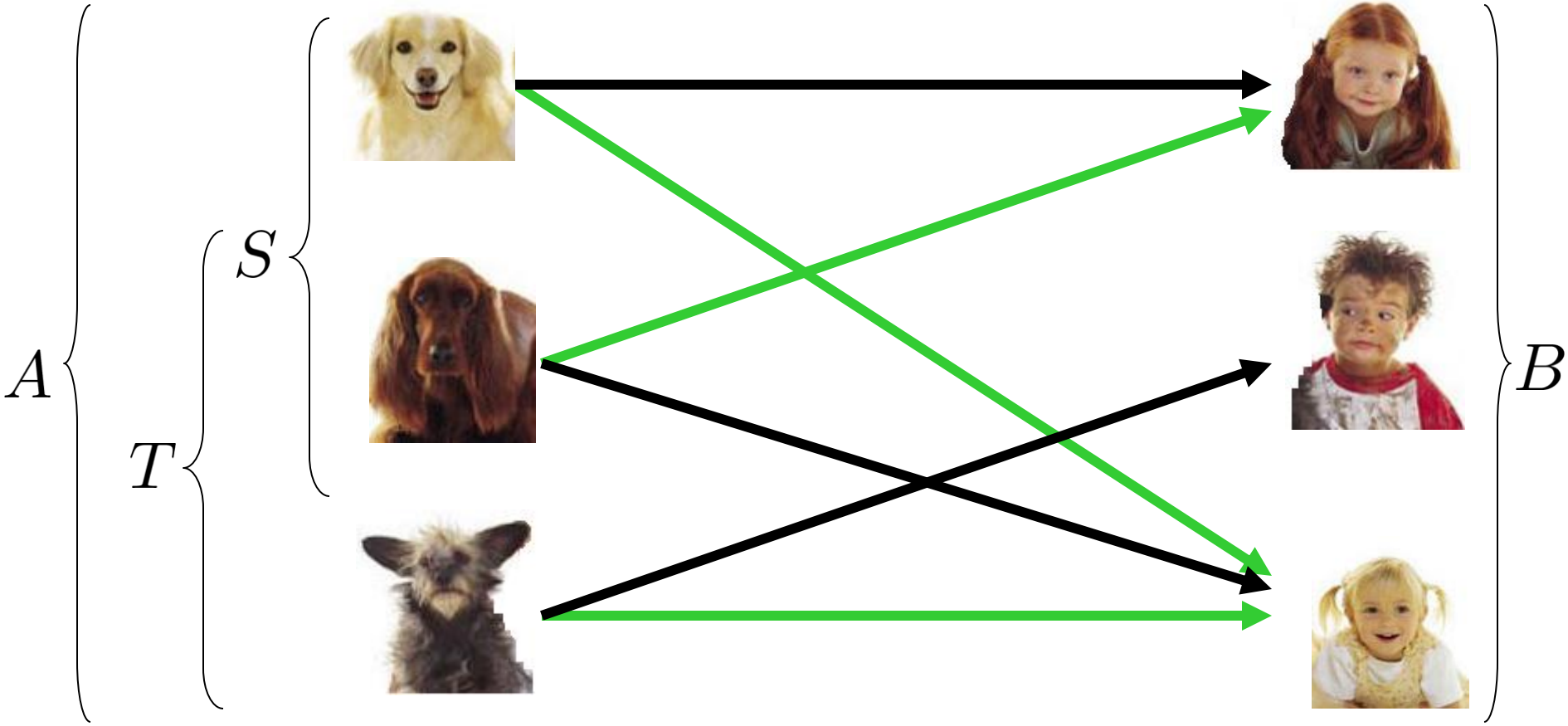

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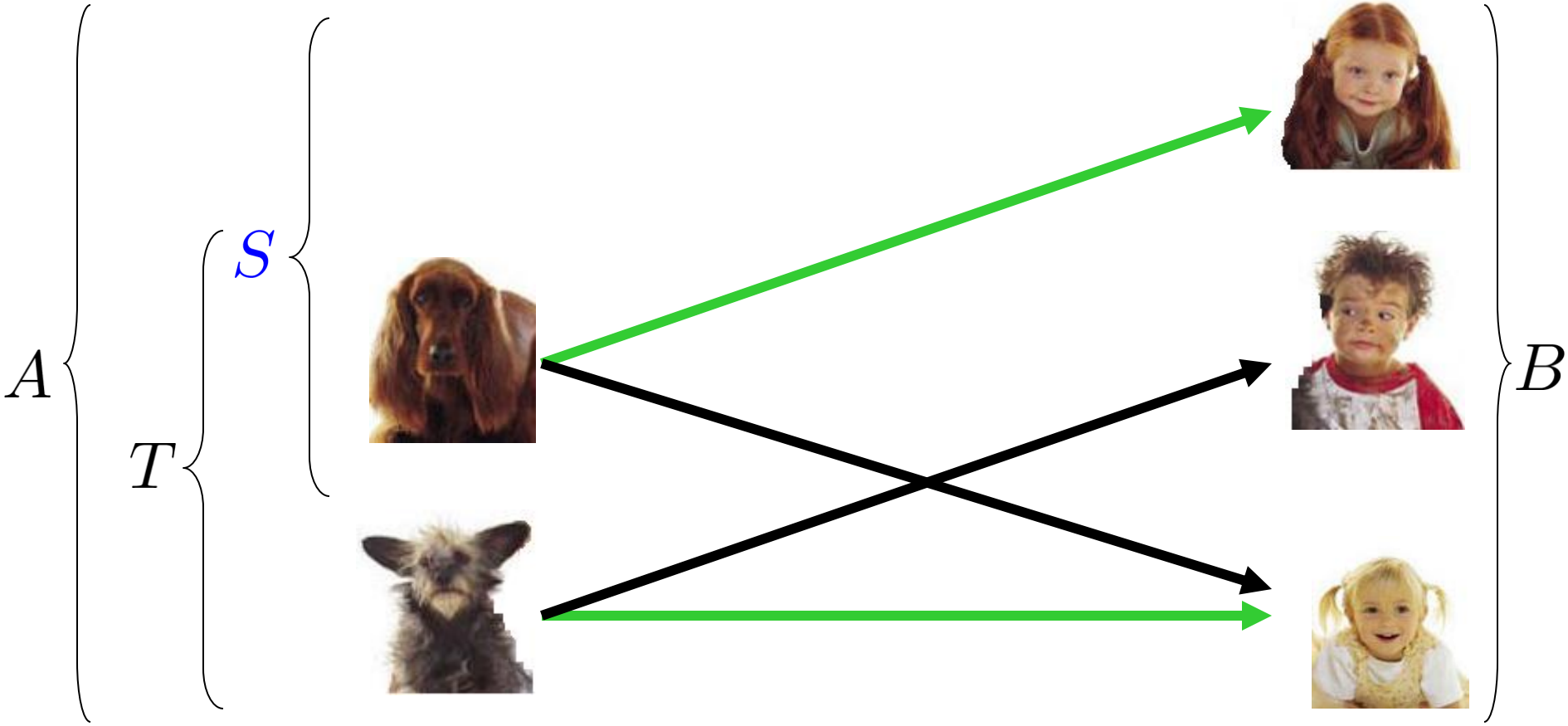

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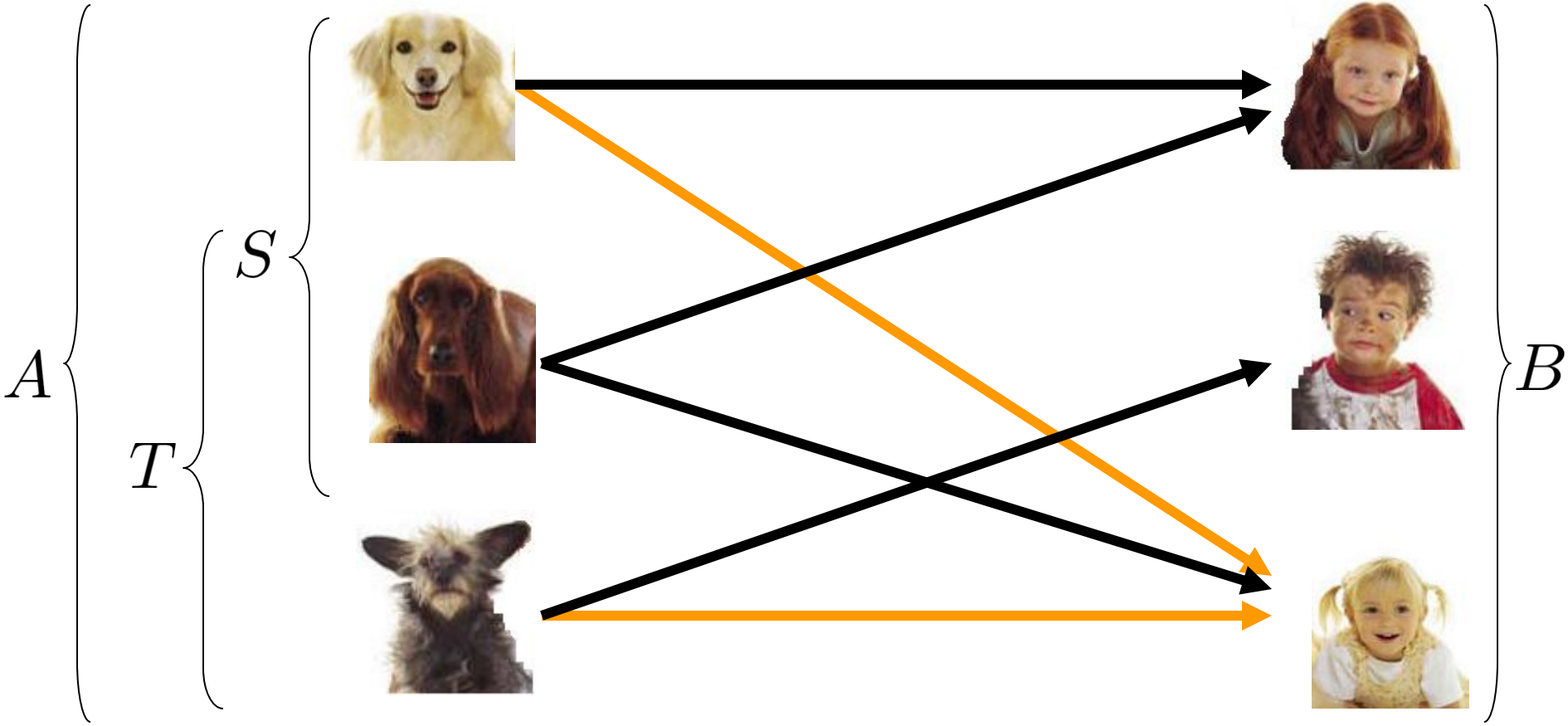

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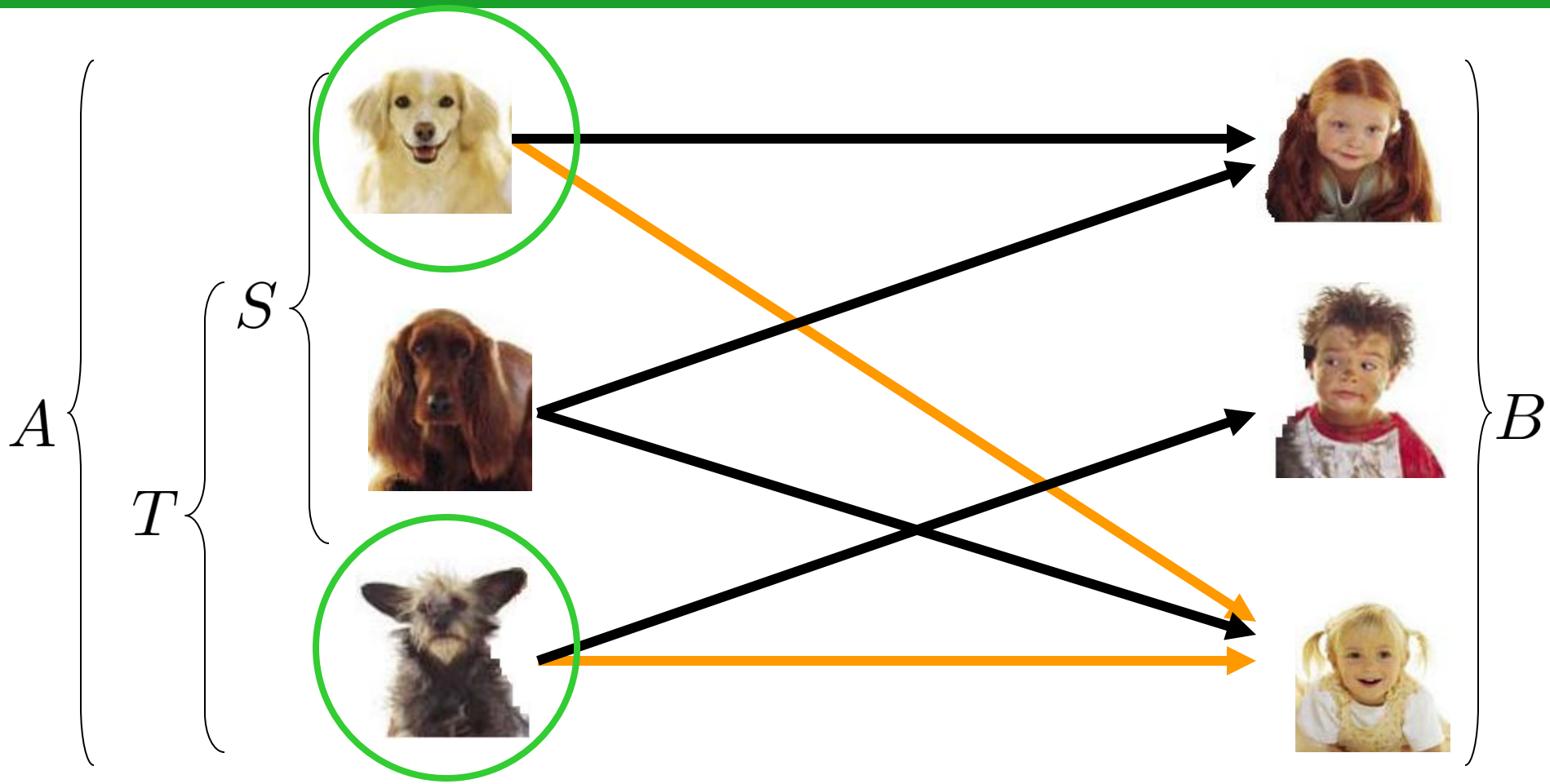


Simultaneous bipartite matching problem

Simultaneous bipartite matching problem.

Simultaneous bipartite matching problem

Let $G = \langle A \cup B, E \rangle$ and S, T be an overlapping bipartite graph. A simultaneous bipartite matching is a set of edges $M \subseteq E$ such that $M \cap (S \times B)$ and $M \cap (T \times B)$ are matchings that cover S and T, respectively.

Simultaneous bipartite matching problem

SBM is NP-complete [2005].

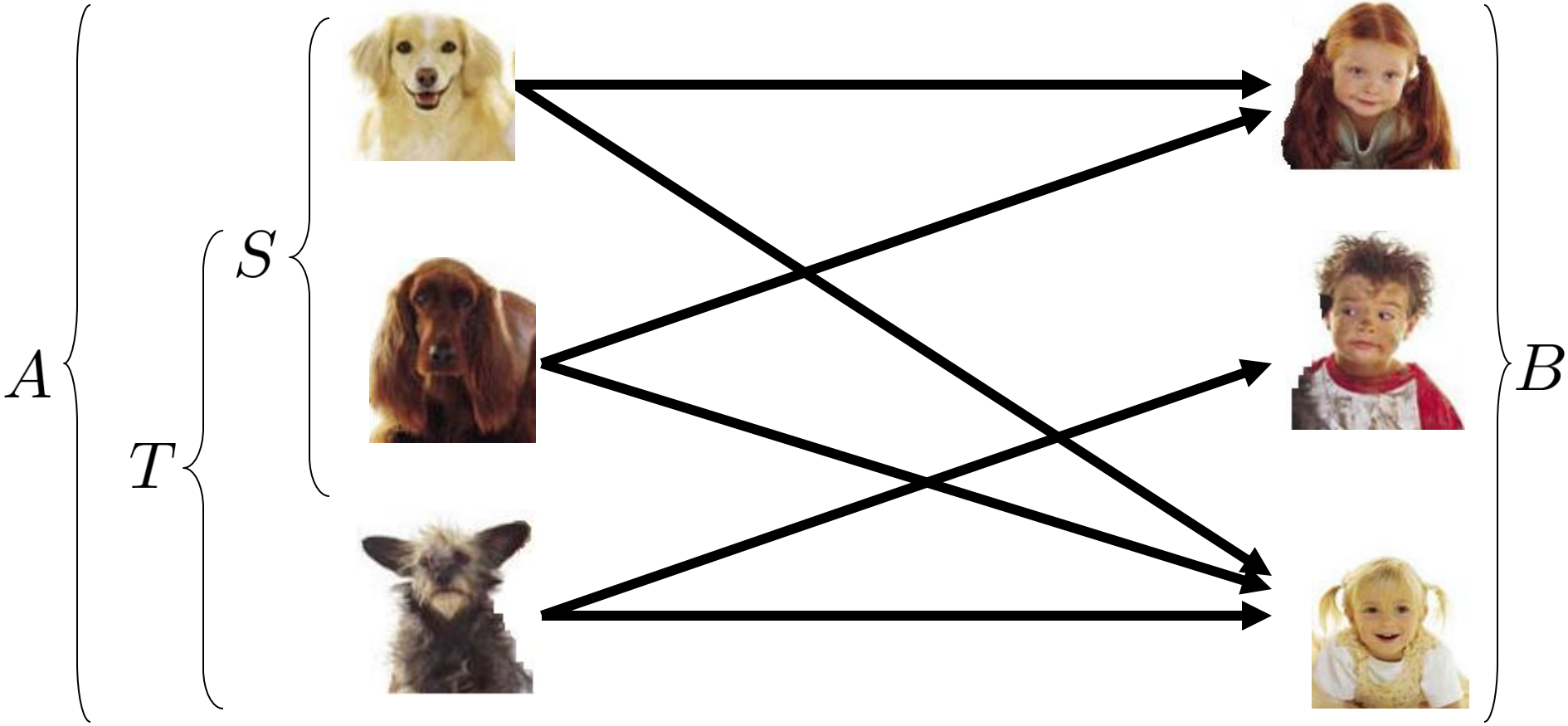
Simultaneous bipartite matching problem

Let $G = \langle A \cup B, E \rangle$ and sets S, T
be an overlapping bipartite graph. There exists a SBM,
iff
$|N(P)| + |N(P_S \setminus P_T) \cap N(P_T \setminus P_S)| \geq |P|$ for $P \subseteq A$.

Simultaneous bipartite matching problem

The problem is NP-complete.
Why are we doing all this work?

Simultaneous bipartite matching problem



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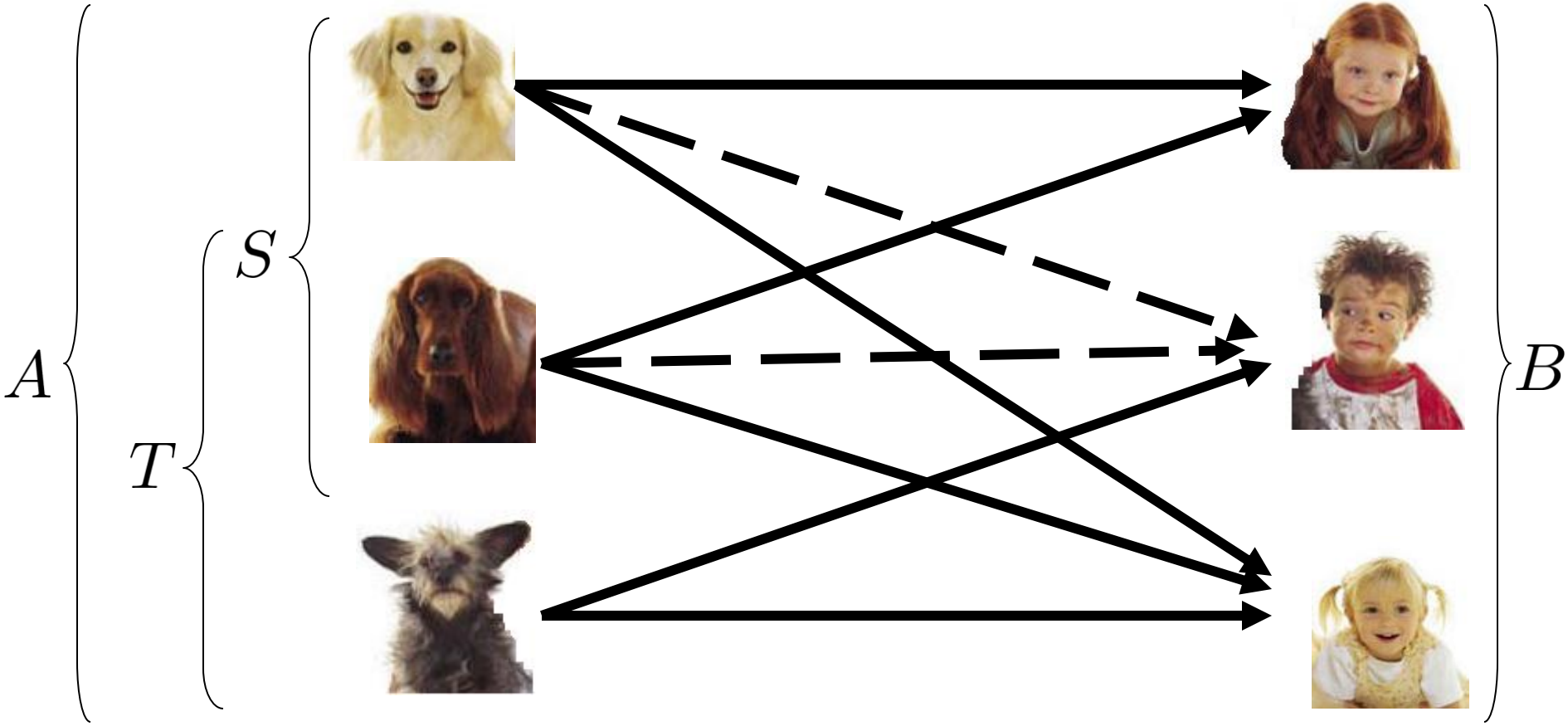

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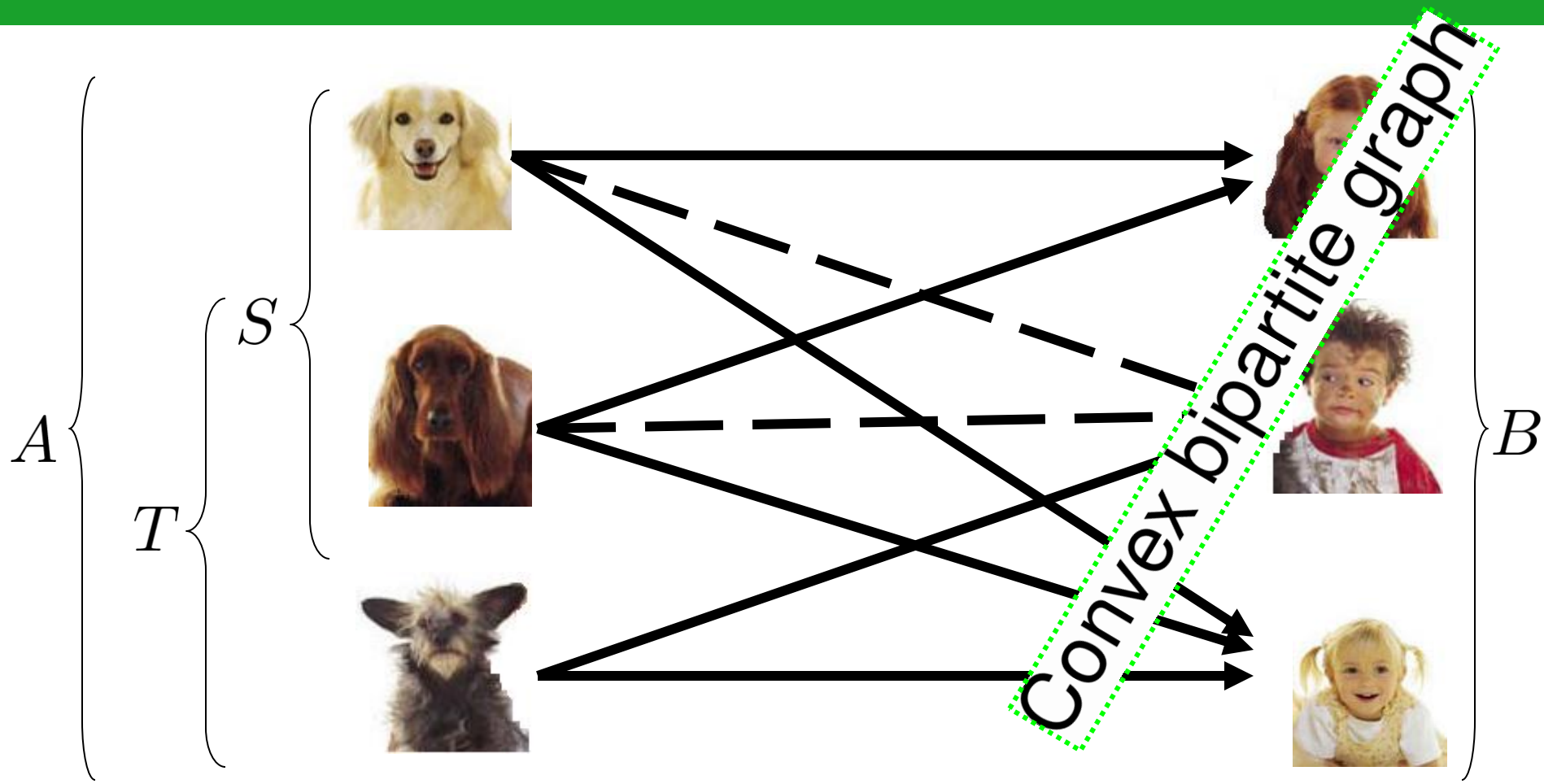


Simultaneous bipartite matching problem

We solved by reformulation
into a system of arithmetic constraints

Simultaneous bipartite matching problem

If the system of arithmetic constraints reaches
a non-empty fixpoint then
the simultaneous Hall condition holds for $\forall P \subseteq A$.

Simultaneous bipartite matching problem

SBM on convex graphs can be solved in $O(V^3)$ time.

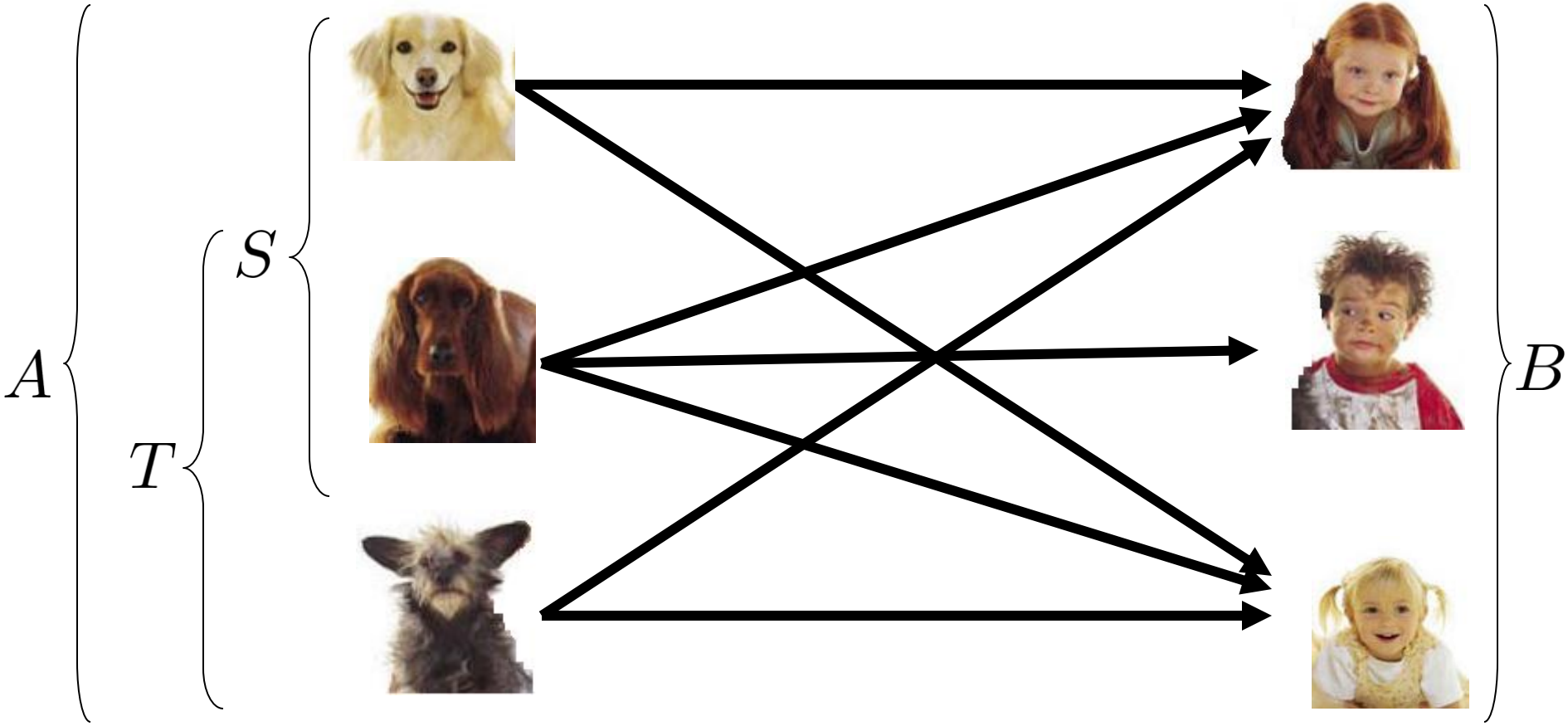
Simultaneous bipartite matching problem

a faster algorithm for convex graphs.

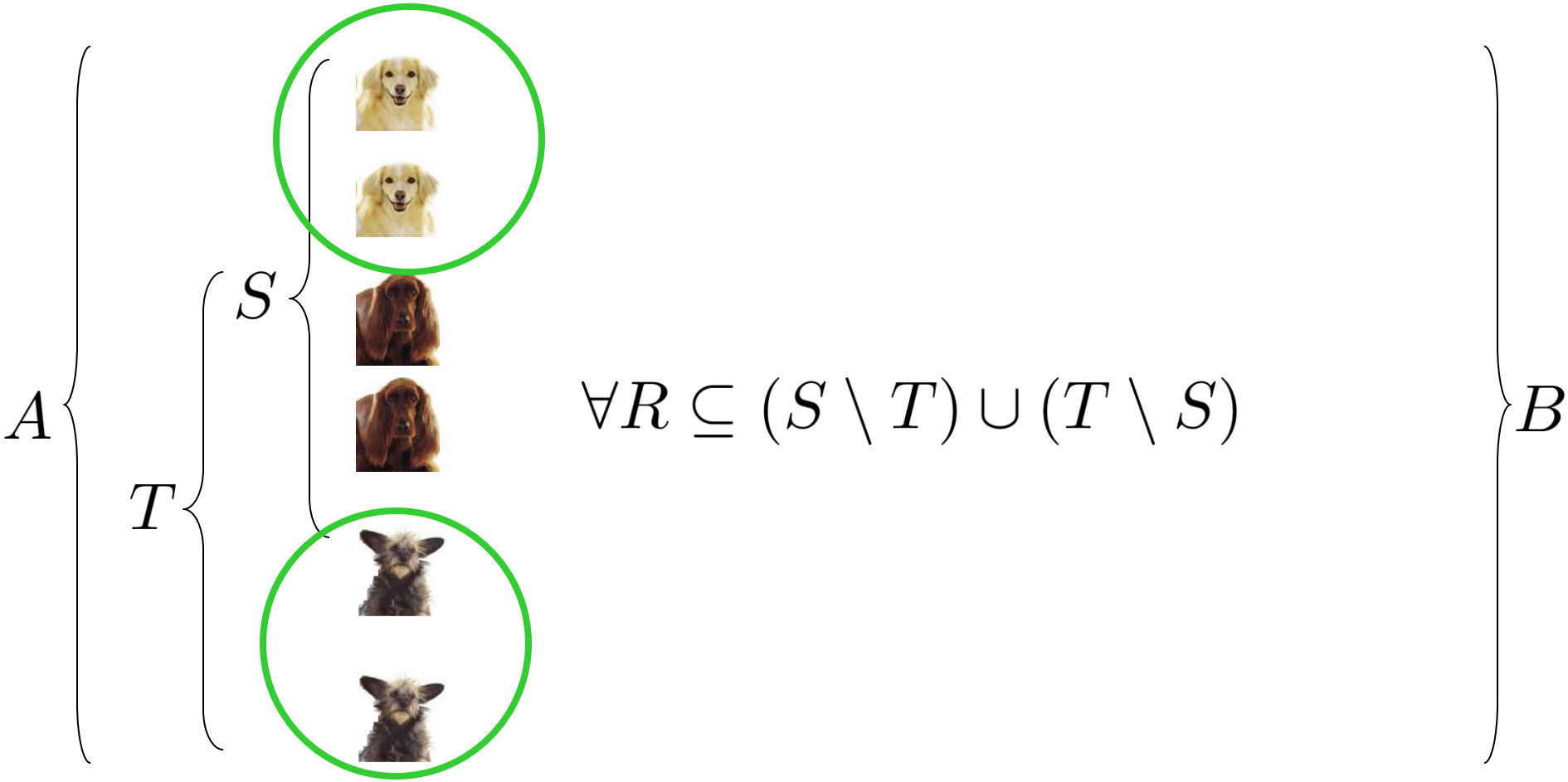
Simultaneous bipartite matching problem

FPT algorithm for general bipartite graphs.

Simultaneous bipartite matching problem

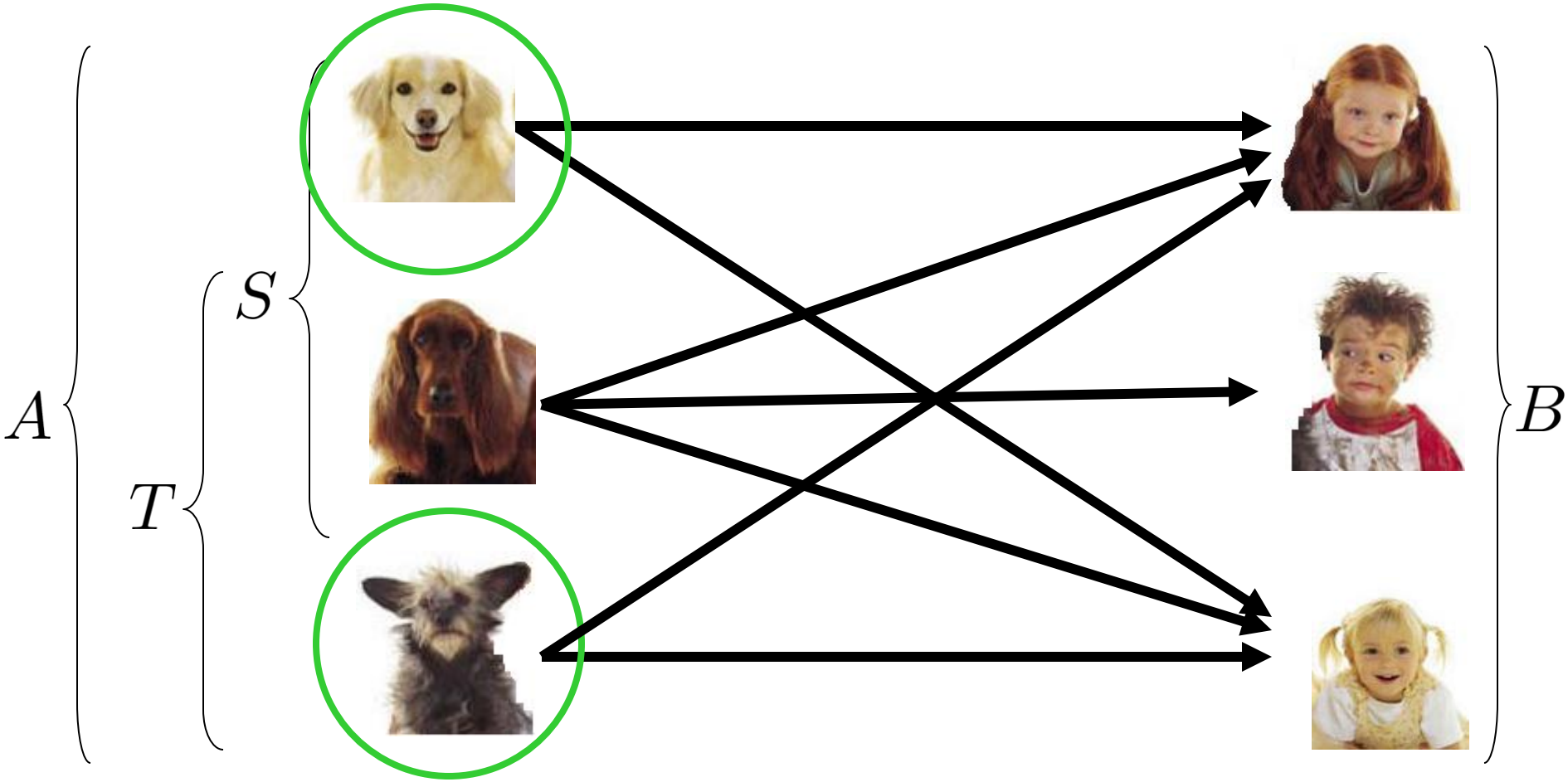


Simultaneous bipartite matching problem



$$\forall R \subseteq (S \setminus T) \cup (T \setminus S)$$

Simultaneous bipartite matching problem



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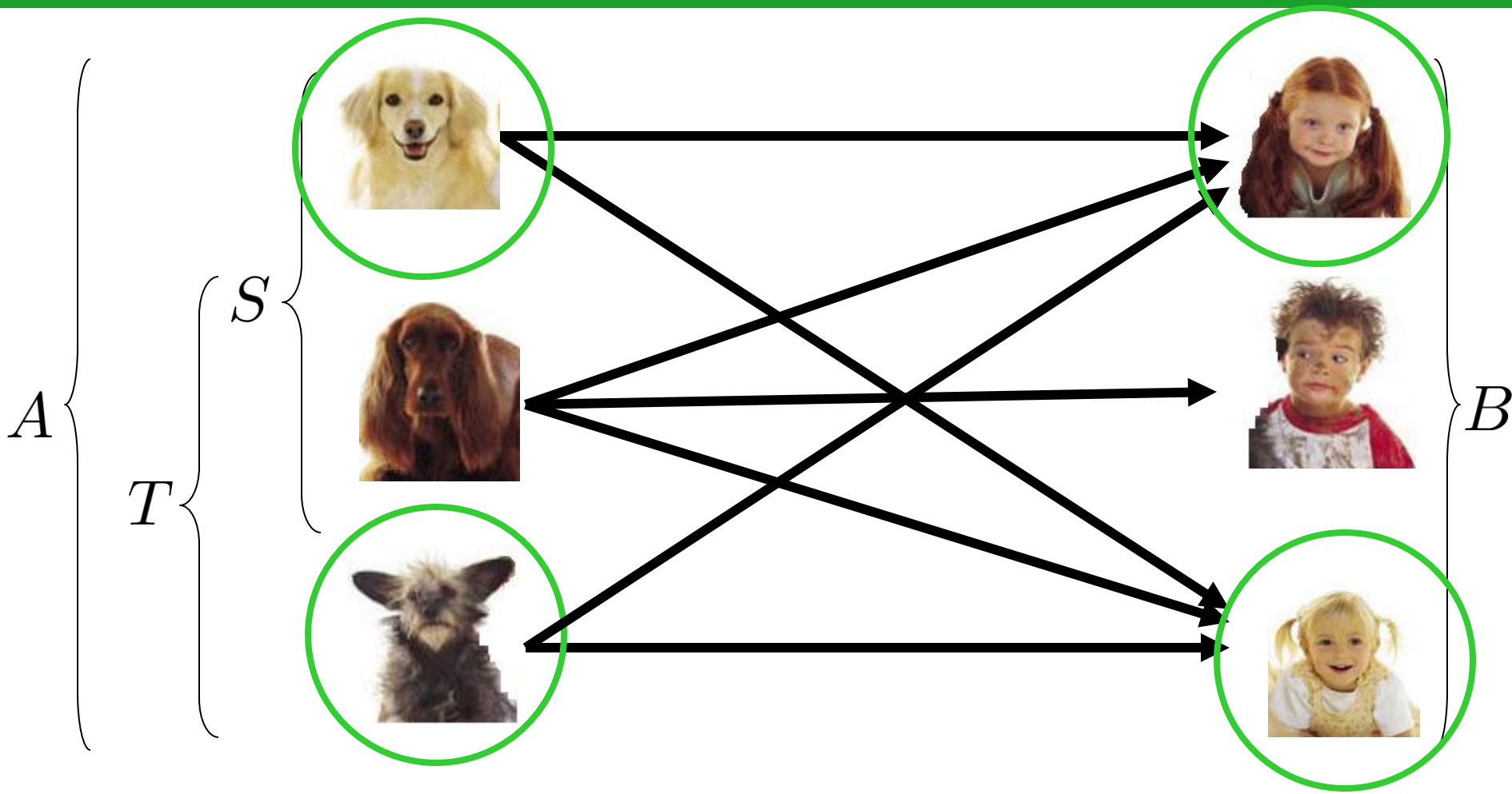

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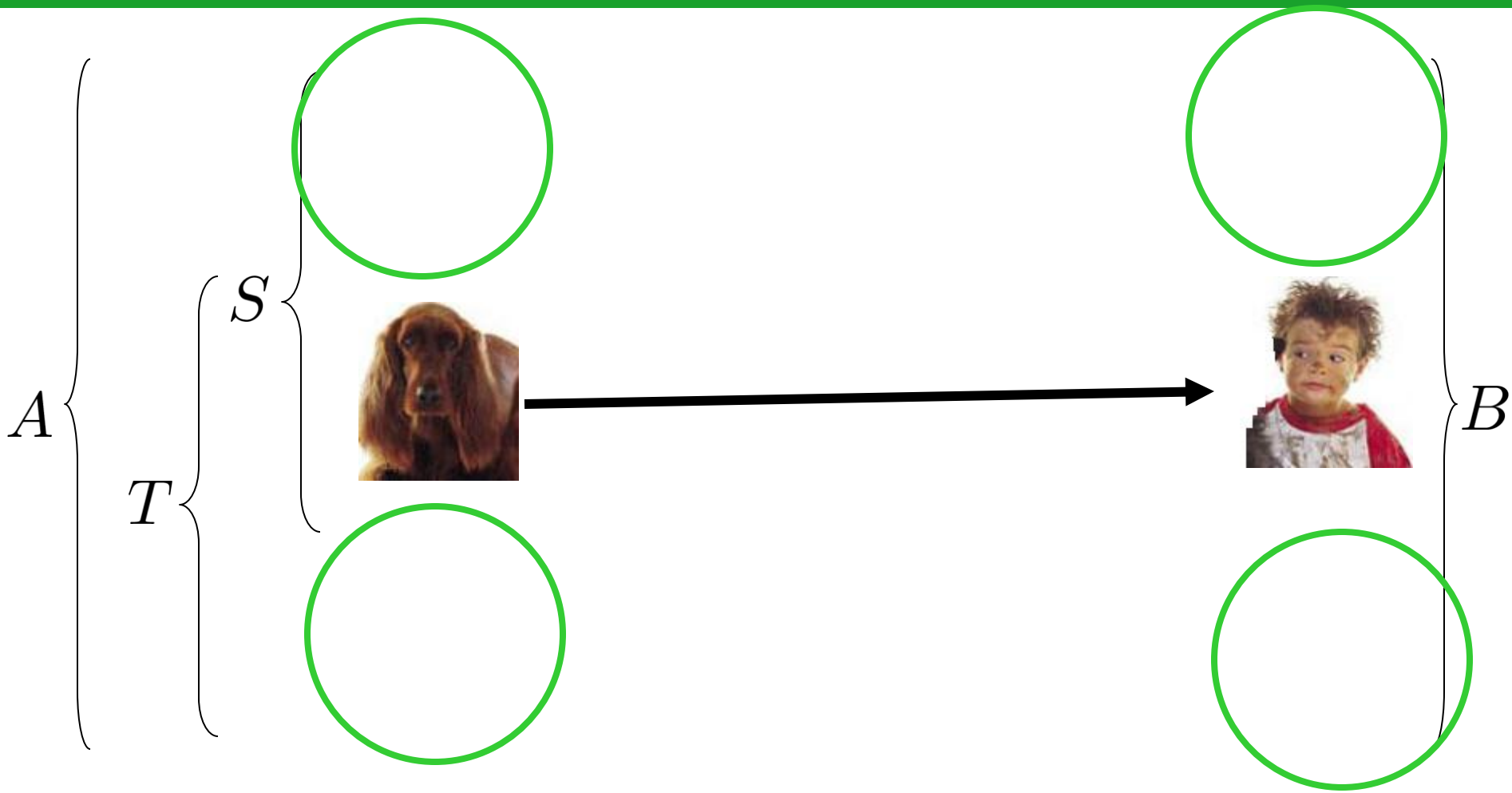

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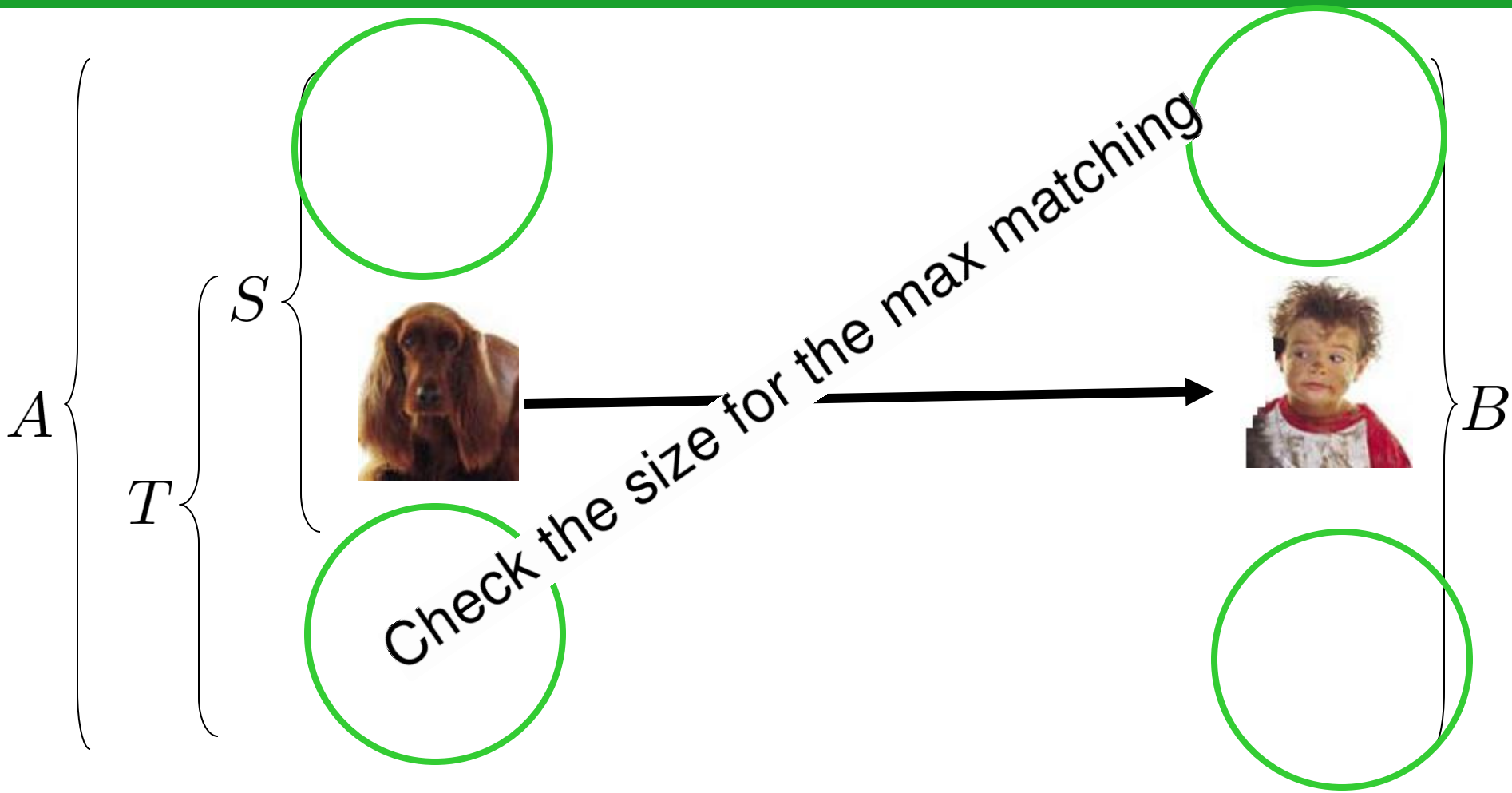

Simultaneous bipartite matching problem

FPT algorithm for general bipartite graphs.

$$O(2^{|S \setminus T|+|T \setminus S|}|S \cup T||E|)$$

Simultaneous bipartite matching problem

Work in progress:

FPT algorithm for general bipartite graphs.

$$O(2^{min(|S\setminus T|,|T\setminus S|)}|S \cup T||E|)$$

Simultaneous bipartite matching problem

We can build a propagator!

FPT algorithm for general bipartite graphs.

$$O(2^{min(|S\setminus T|,|T\setminus S|)}|S \cup T||E|)$$

FPT for Multiple AllDifferent Constraint

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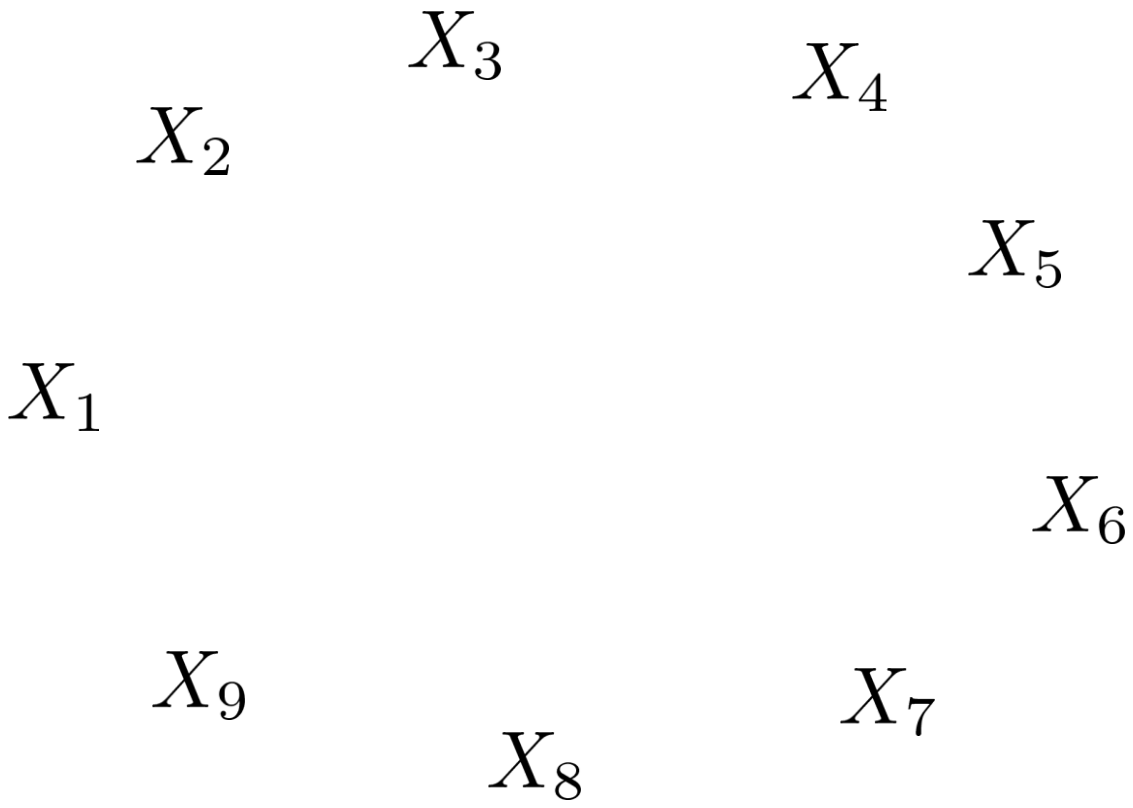
Constraint graph



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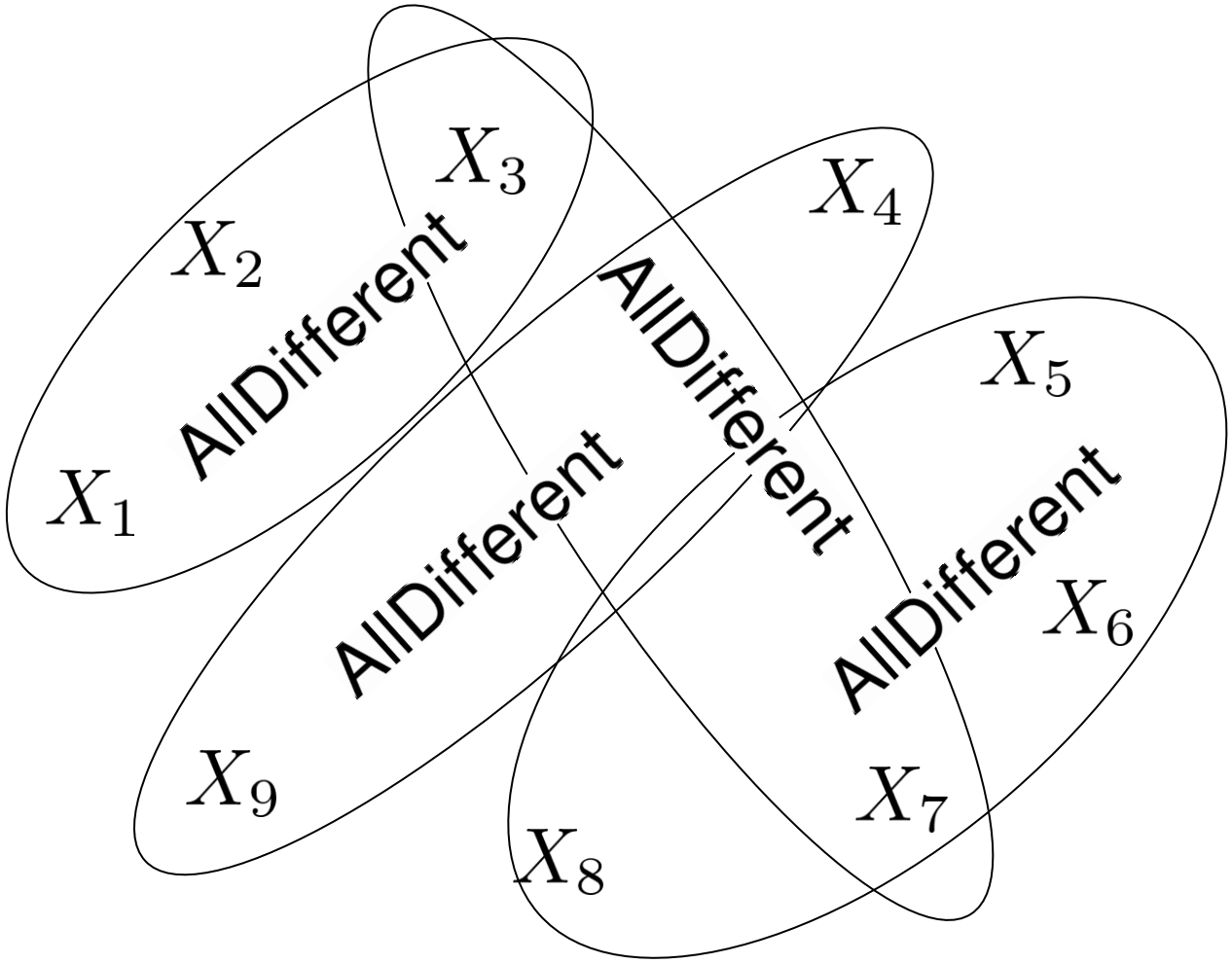

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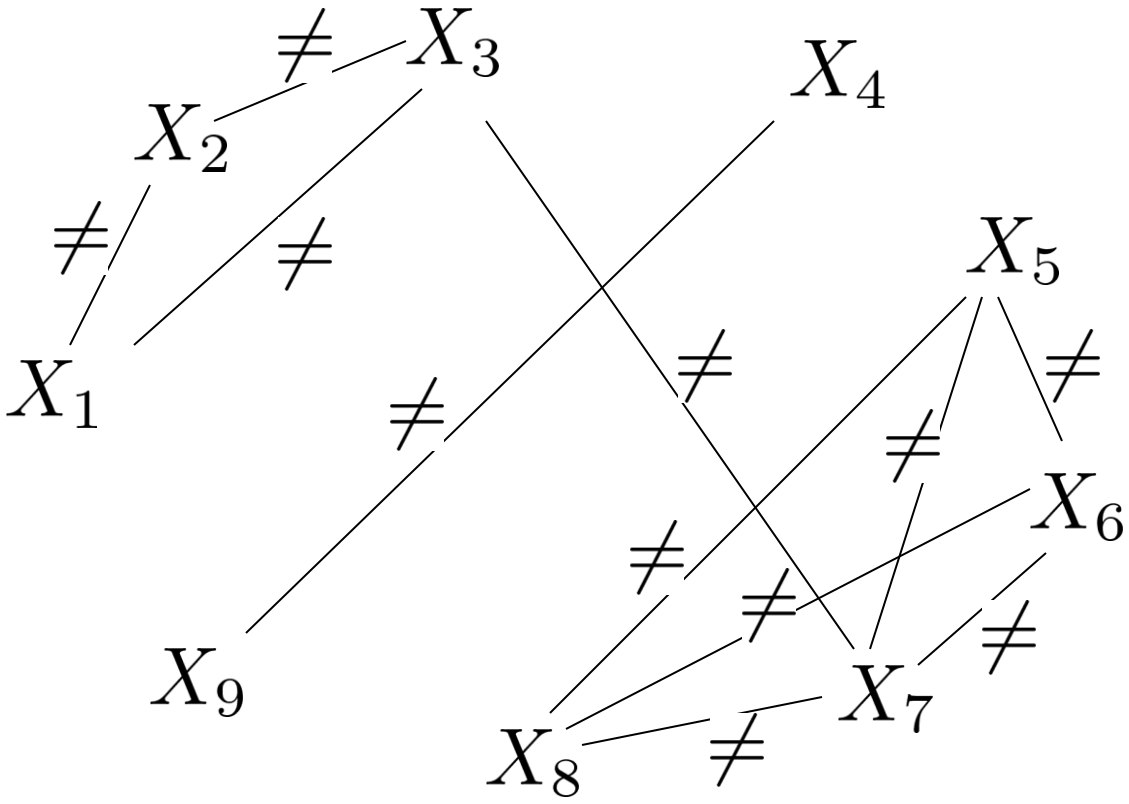

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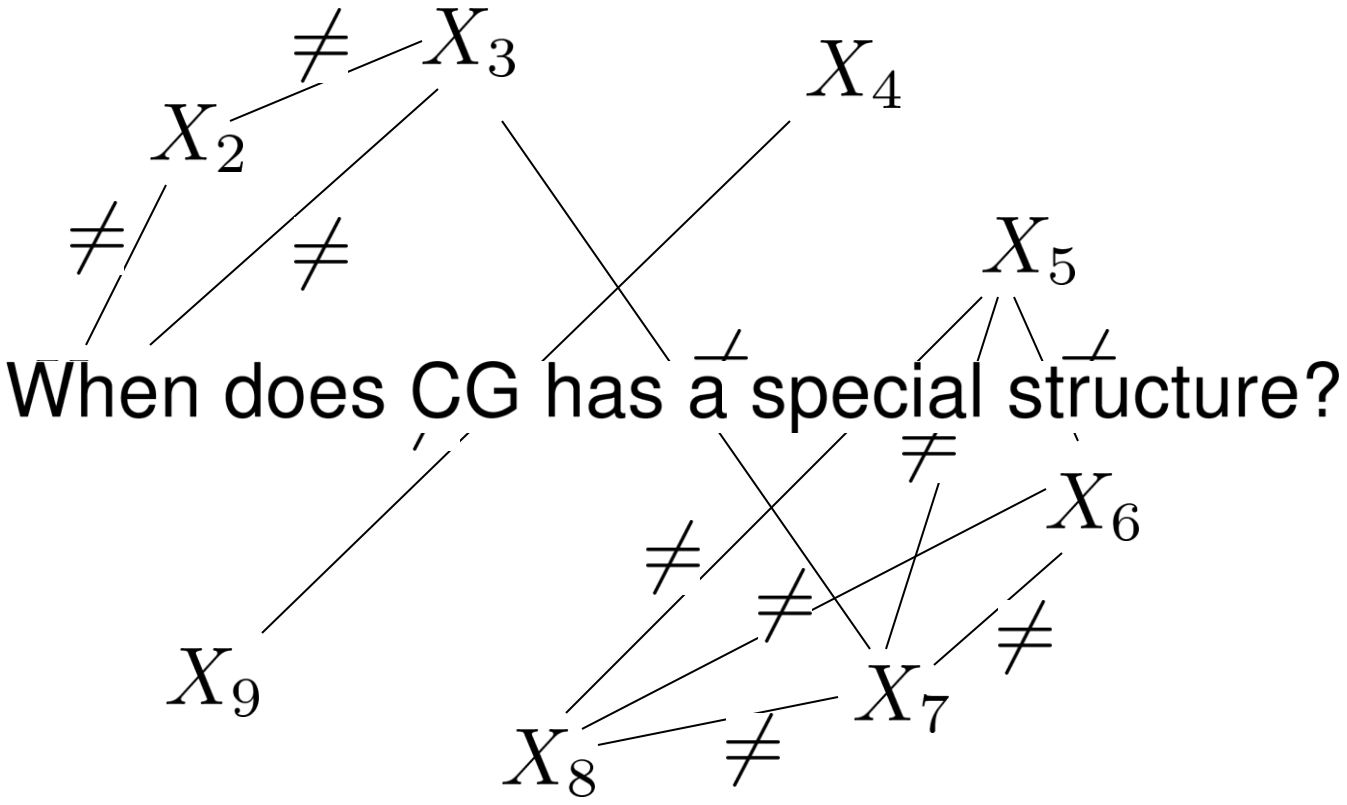
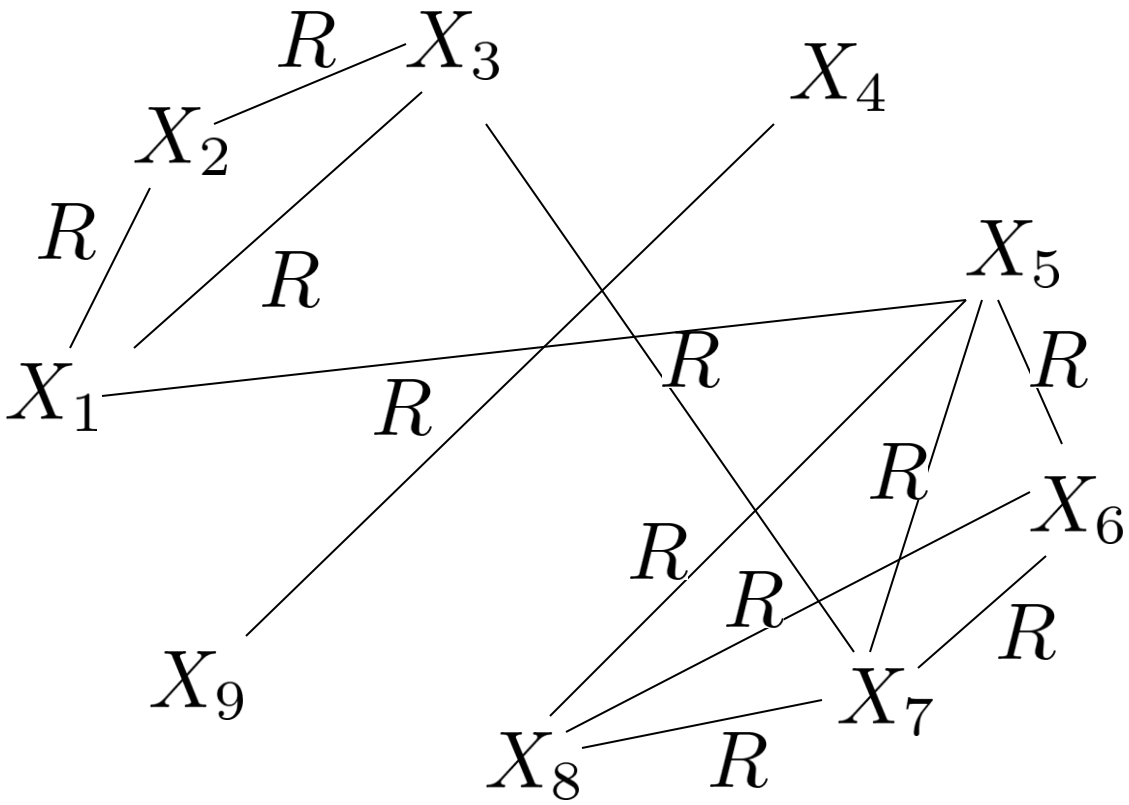


Constraint graph

Bad news

Constraint graph



Same-Relation Constraint [CP2009]

Constraint graph

1. Clique
2. Bipartite graph
3. Grid

Same-Relation Constraint [CP2009]

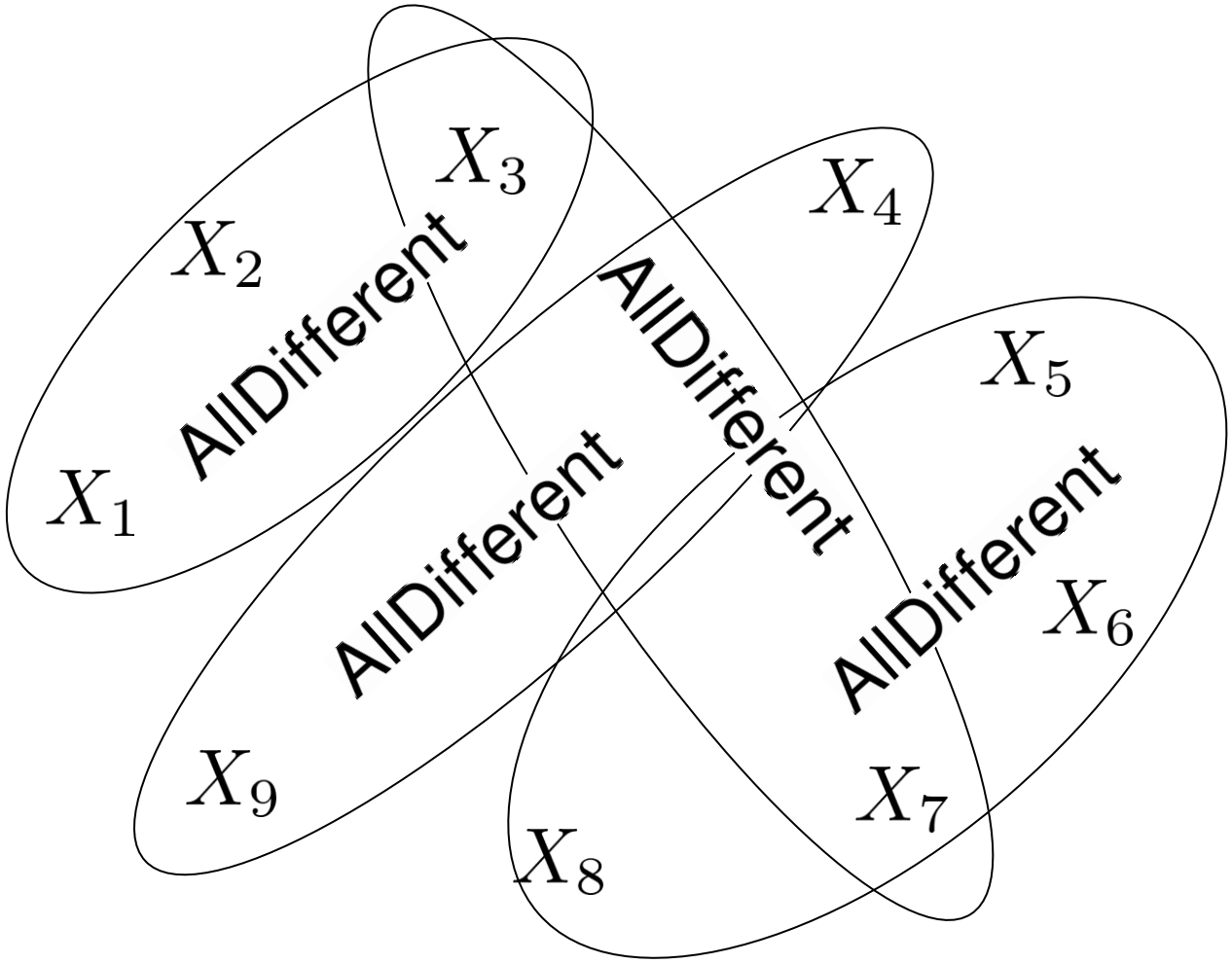
Constraint graph

Good news

Constraint graph

1. CG is induced by MultipleAllDifferent
2. R is $\neq$
3. assumptions on variables domains
4. assumptions on costraints overlaps

Constraint graph



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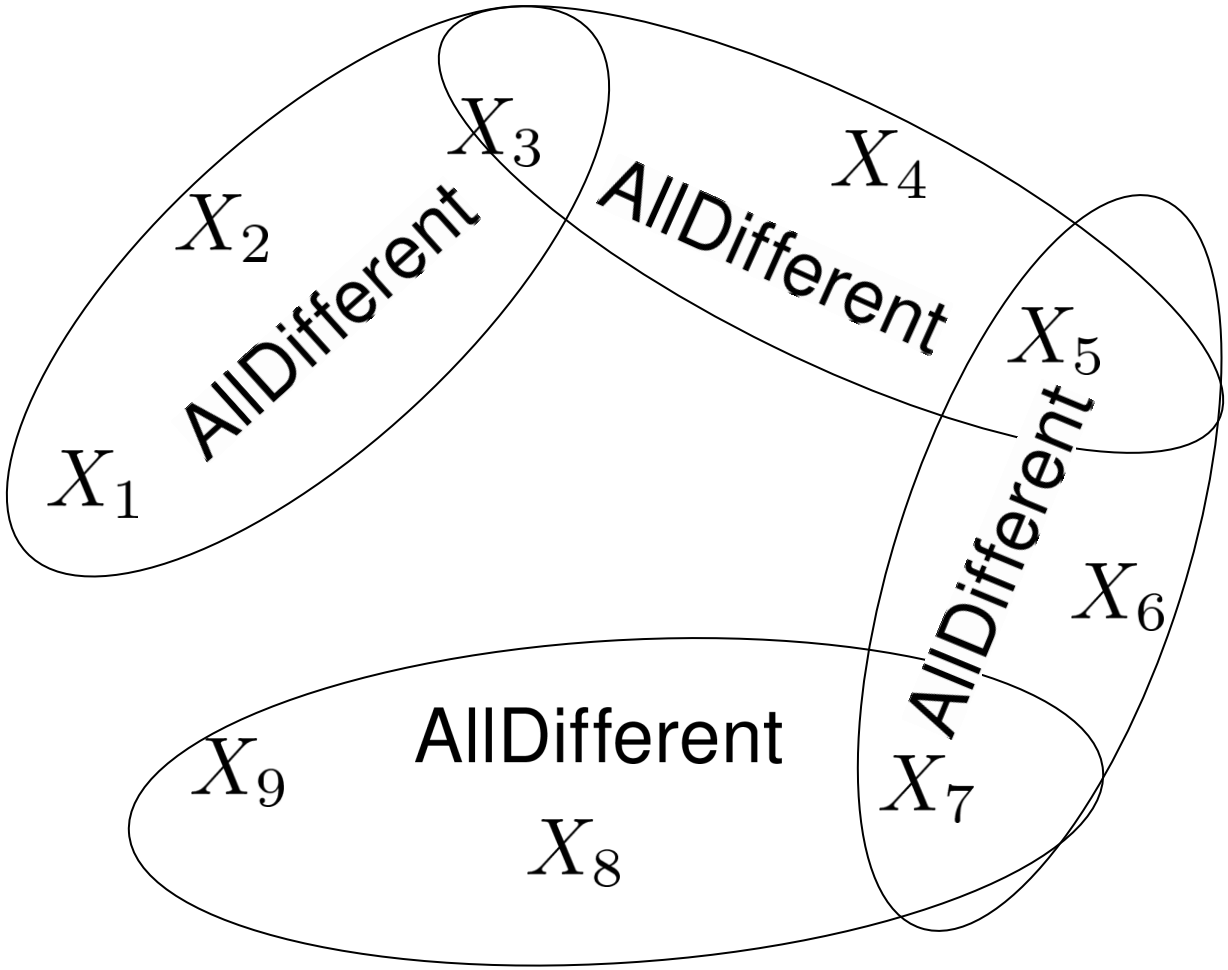
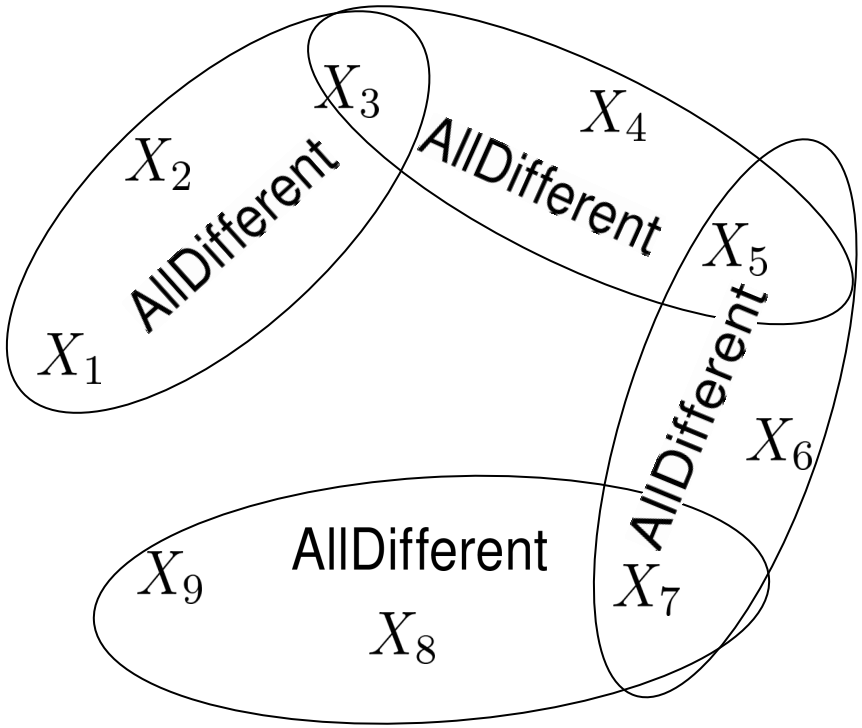


Constraint graph

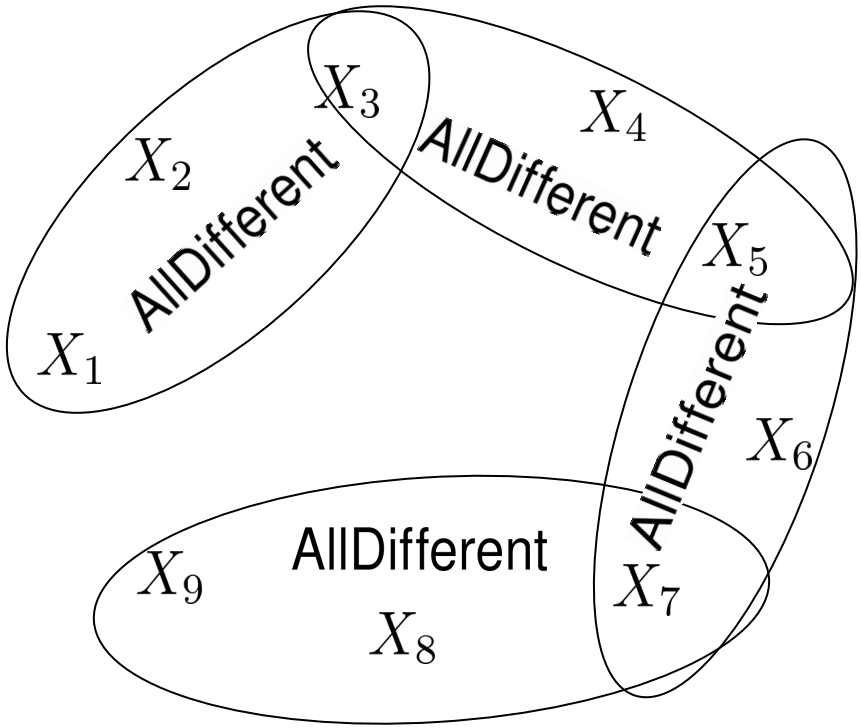
1. consecutive cons



Constraint graph

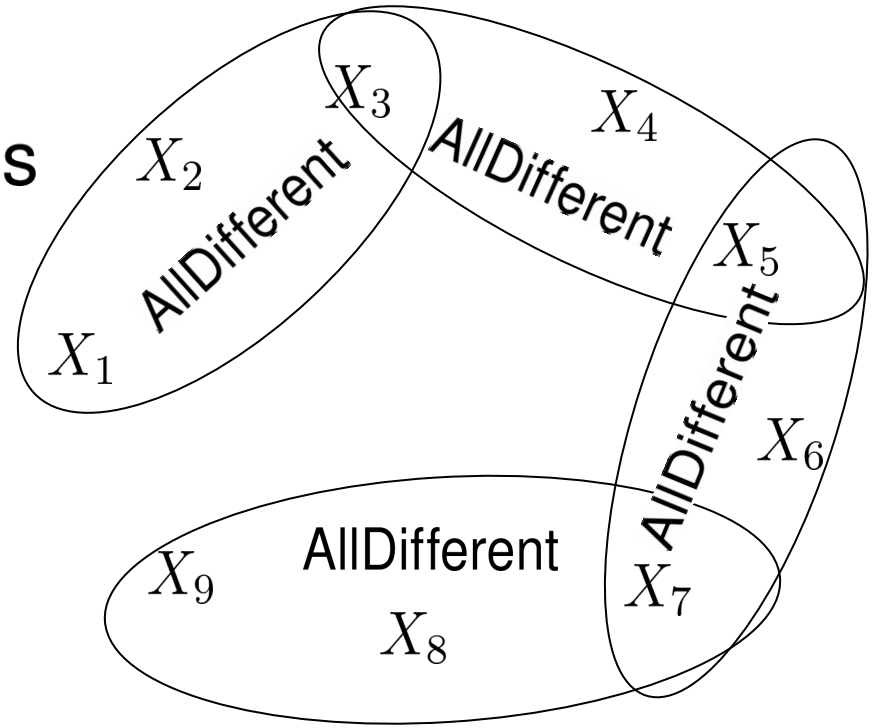
1. consecutive cons

NP-complete for 2 cons



Constraint graph

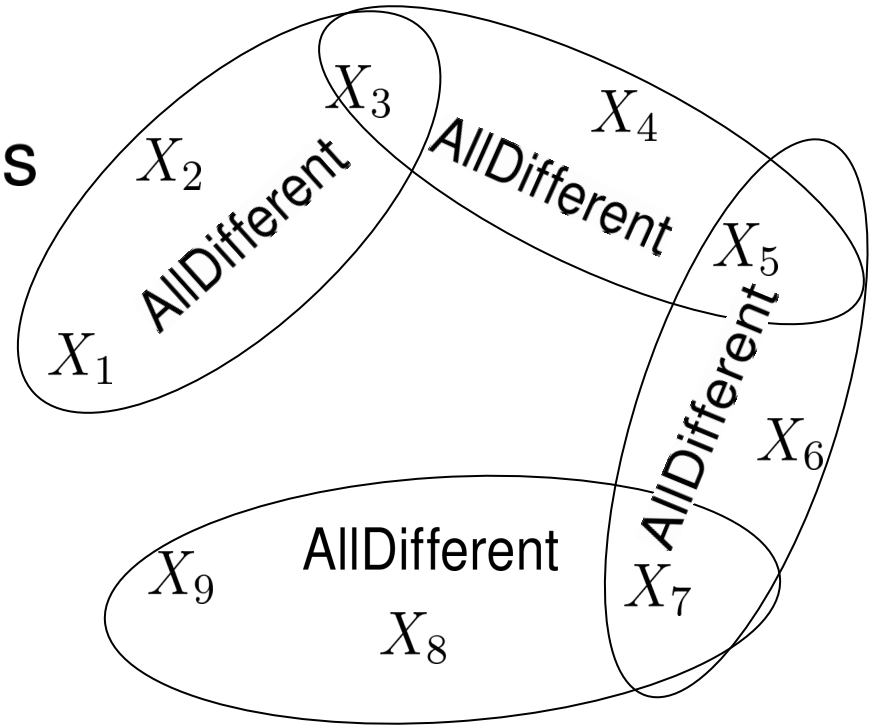
1. consecutive cons
2. interval variables domains



Constraint graph

1. consecutive cons
2. interval variables domains

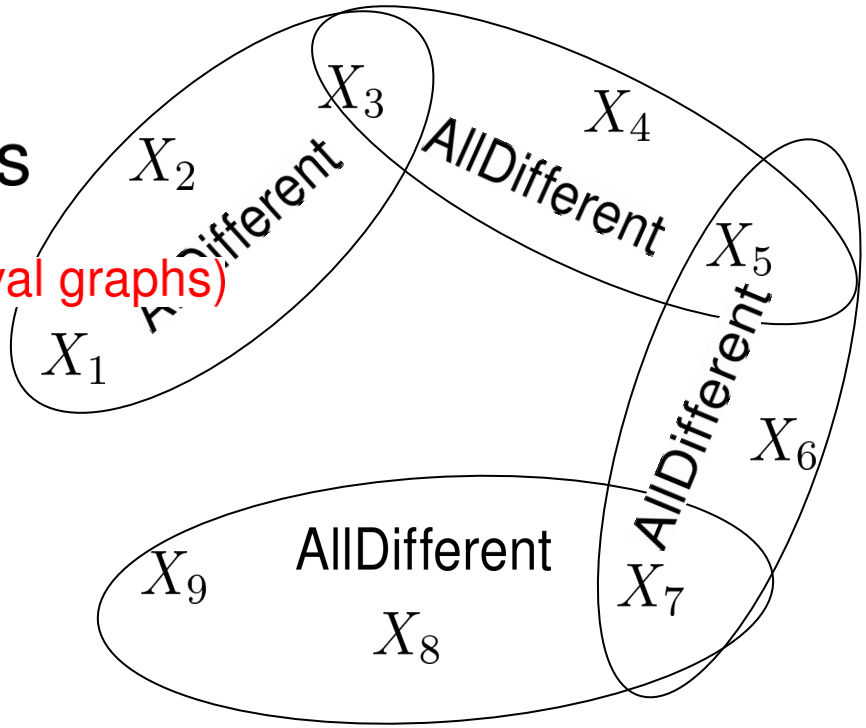
   P for 2 cons



Constraint graph

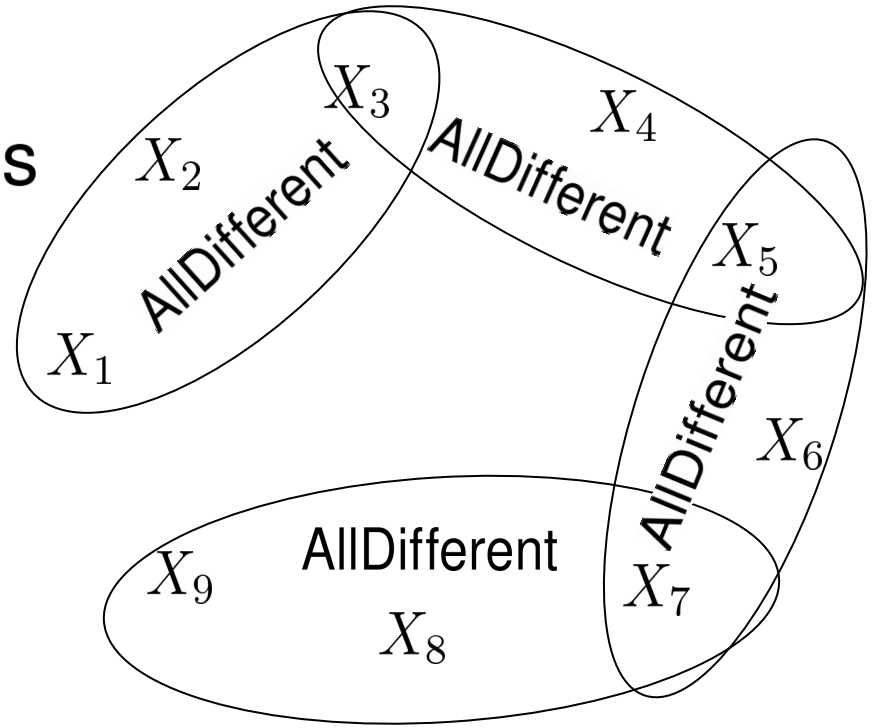
1. consecutive cons
2. interval variables domains

NP-complete (α, β-list coloring on intreval graphs)



Constraint graph

1. consecutive cons
2. interval variables domains
3. bounded arity of cons



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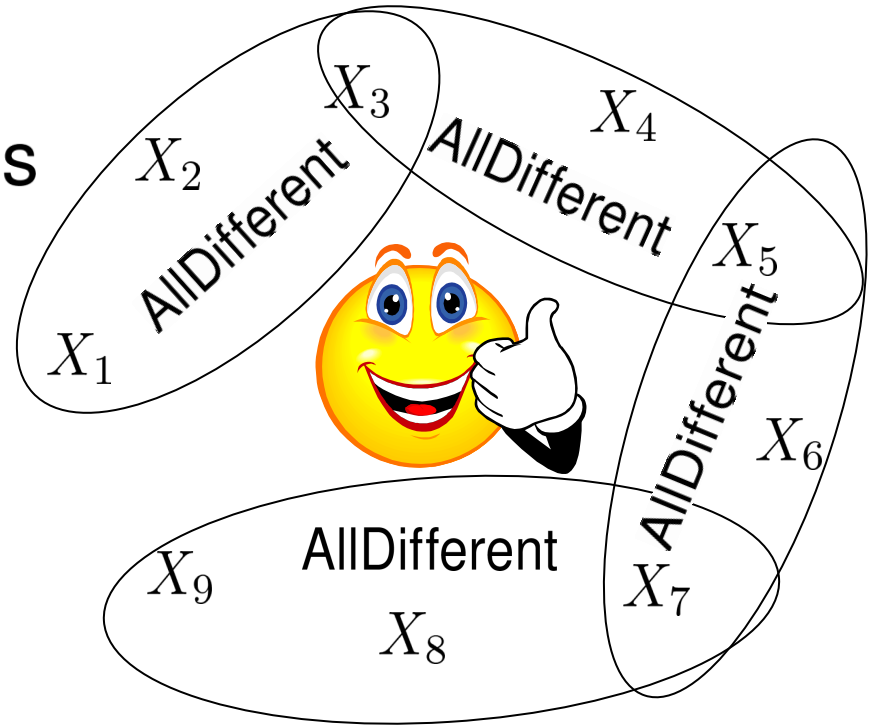

Constraint graph

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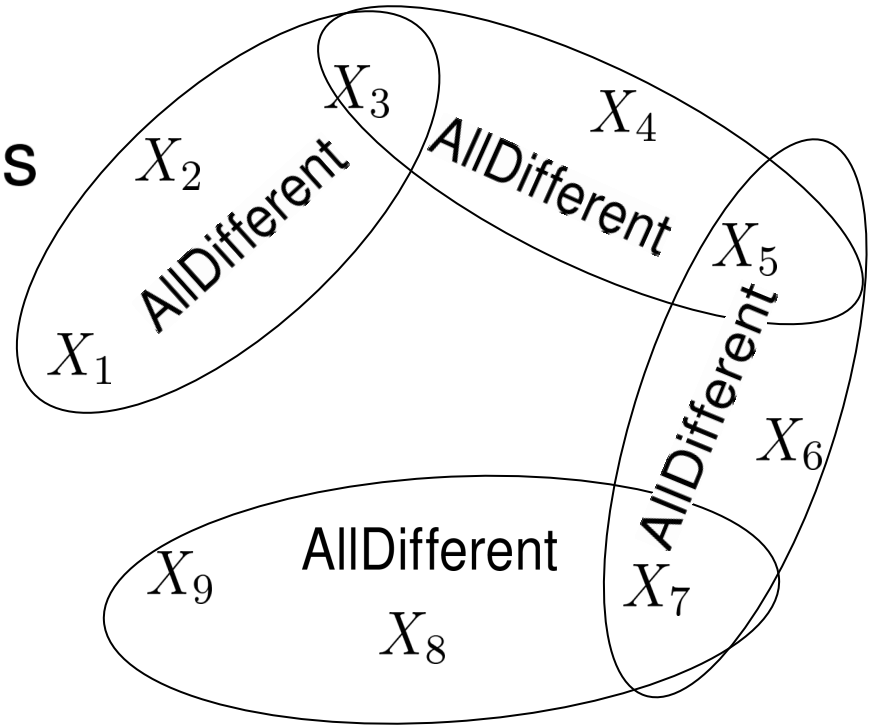
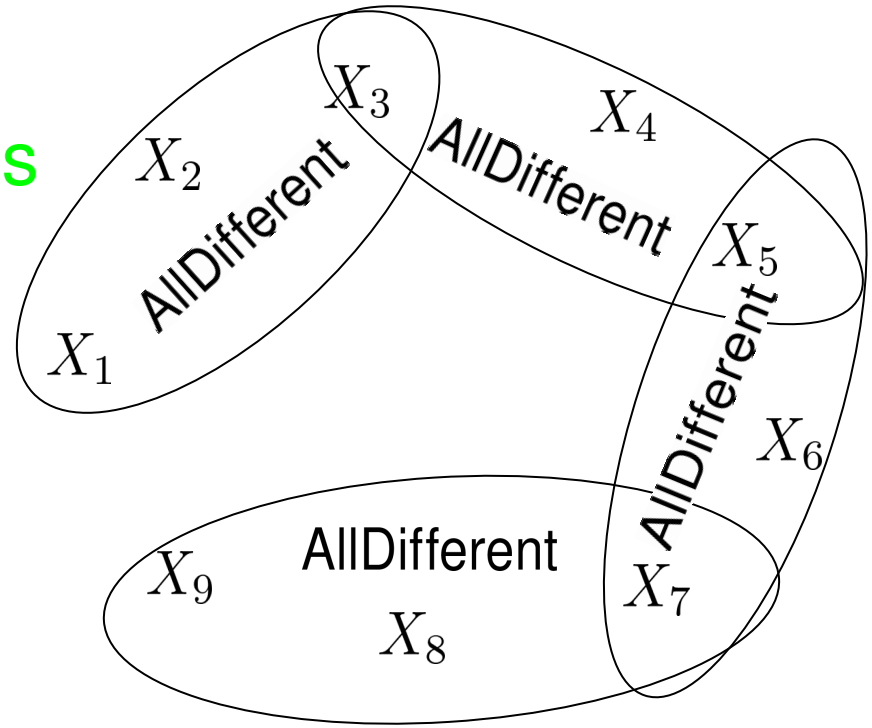


FPT: the paramater is the max constraint arity

Constraint graph

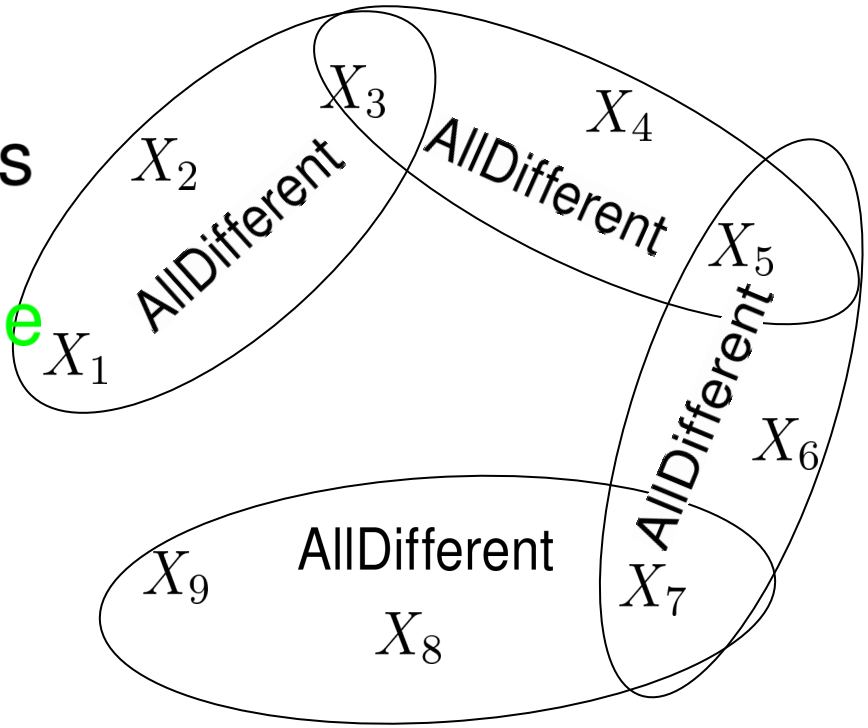
1. consecutive cons
2. interval variables domains
3. bounded arity of cons



FPT: the paramater is the max constraint arity

Constraint graph

1. consecutive cons
2. interval variables domains
3. bounded max domain size



FPT

Thank you!