On an odd case of an XP algorithm for graphs of bounded clique-width

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Based on joint work with
R. Ganian and J. Obdržálek,
orig. presented at STACS 2011.

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can one always process those throughout the recursion?

More Formally...

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Canonical equivalence; metadefinition Abrahamson–Fellows

The *canonical equivalence* of $\mathcal{P}$ on the universe $\mathcal{U}$ is defined:

$(G_1, \varphi_1) \approx_{\mathcal{P}} (G_2, \varphi_2)$ if and only if, for all (H, φ),

$$(G_1, \varphi_1) \otimes (H, \varphi) \models \mathcal{P} \iff (G_2, \varphi_2) \otimes (H, \varphi) \models \mathcal{P} \,.$$

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Definition – cf. the canonical equivalence of $\mathcal{P}$

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Consistency with $\otimes$? – stronger than just "honoring $\approx_{\mathcal{P}}$"

A canonical partition $\mathcal{X}$ is *consistent with* $\otimes$ if

- for all $(G_1, \varphi_1) \in X_1$, $(G_2, \varphi_2) \in X_2$;
  the part of $(G_1, \varphi_1) \otimes (G_2, \varphi_2)$ depends only on X_1, X_2.

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  Hopefully *polynomial table size* $\rightarrow$ hoping for an XP algorithm.
- Table update (wrt. $\otimes$) has to be done in polytime...
- But what if $\mathcal{X}$ is inconsistent with $\otimes$ (i.e., cannot do table update), and we have no "better" canonical partition?

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- Seems to resist nearly all useful parametrizations (except by *clique-width / rank-width*).
- In MSO_2, only one "$\exists F$" above MSO_1.
  Hence if an extension of Courcelle–Makowsky–Rotics is sought (MSO_1 on graphs of bounded clique-width), then MinLOB should be understood first…

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- An "across" edge / arc depends only on the label of its end(s).

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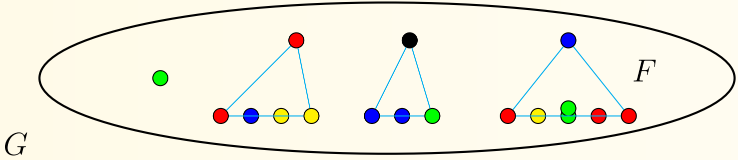

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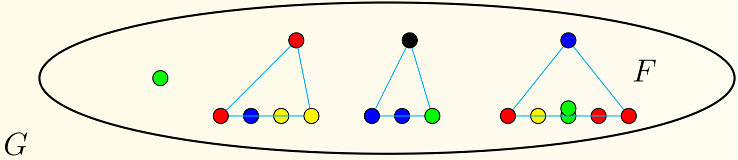

Fact. Strong signatures give canonical classes for MinLOB.

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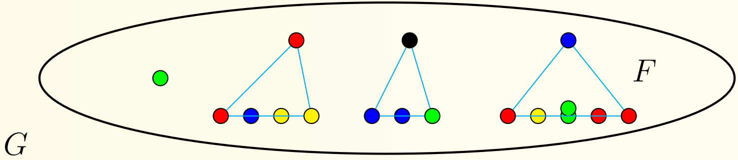

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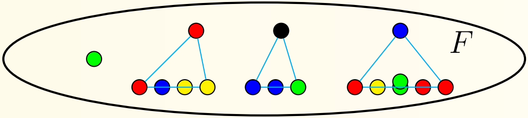

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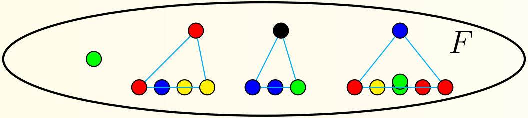

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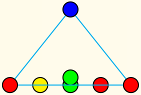
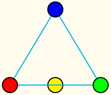
$\rightarrow$ i.e.,  $\longleftrightarrow$  of fixed size(!),

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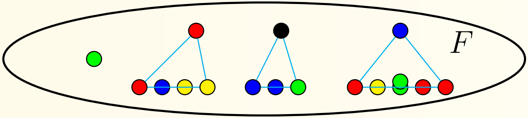

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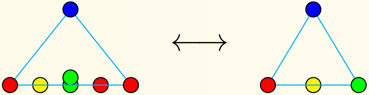
$\rightarrow$ i.e.,  of fixed size(!),

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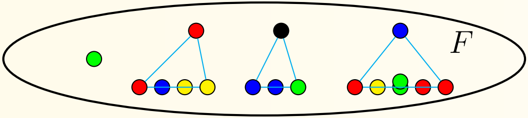

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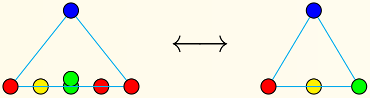
$\rightarrow$ i.e.,  of fixed size(!),

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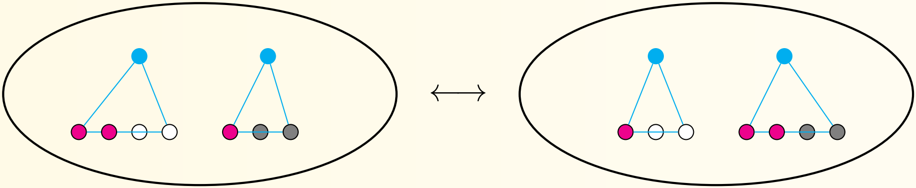

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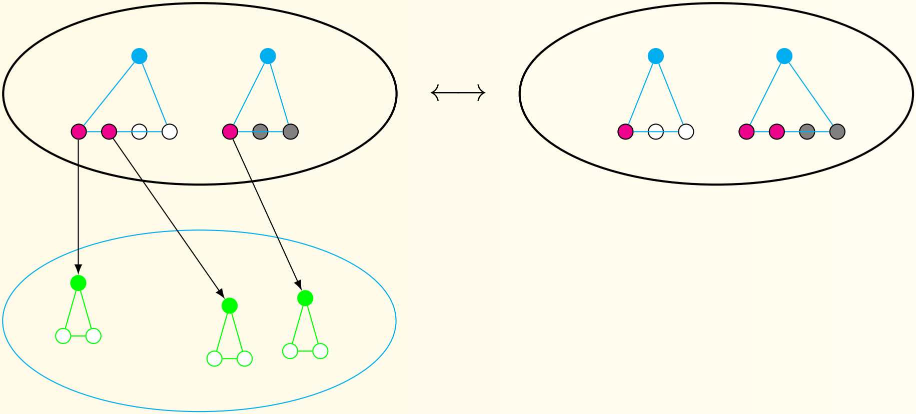

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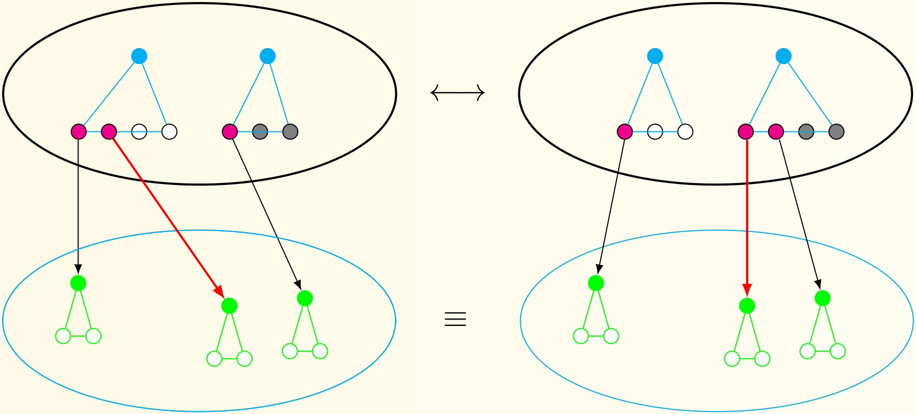

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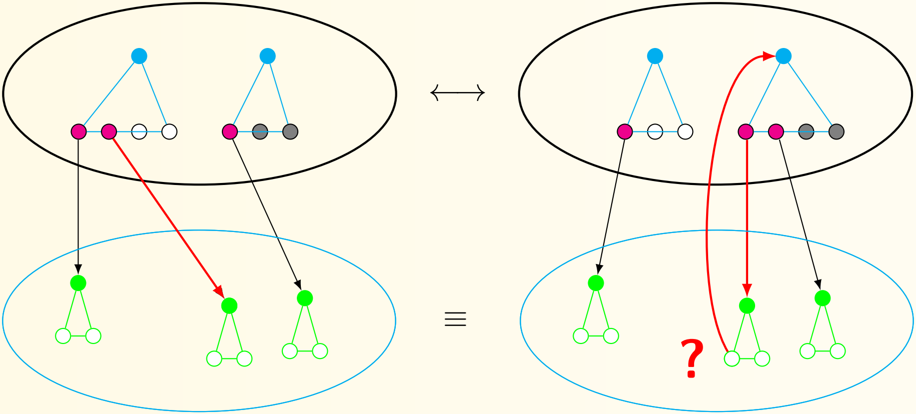

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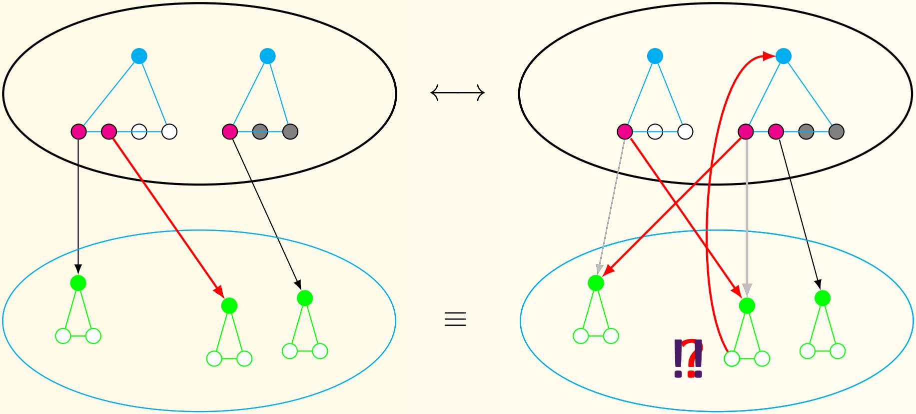

Hence the upper two fragments are canonically equivalent, indeed. □

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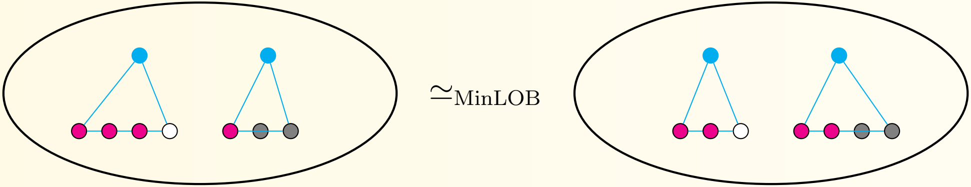

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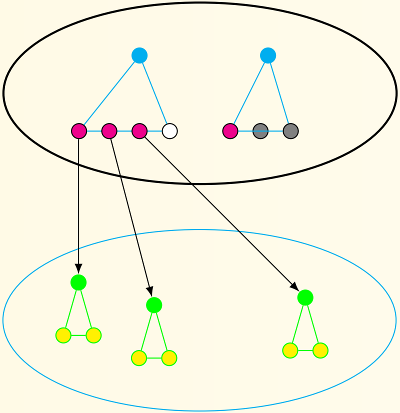
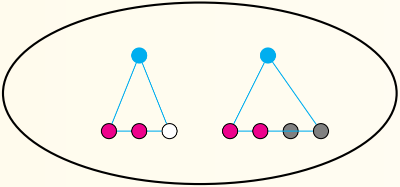


$\simeq_{\mathrm{MinLOB}}$



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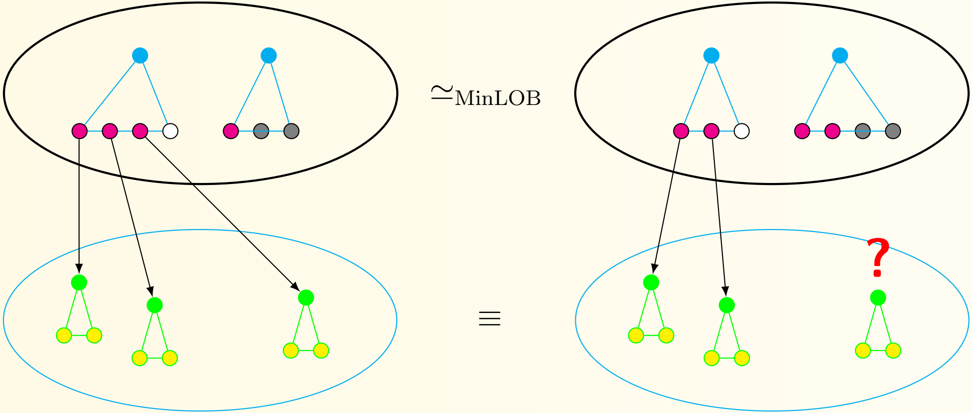

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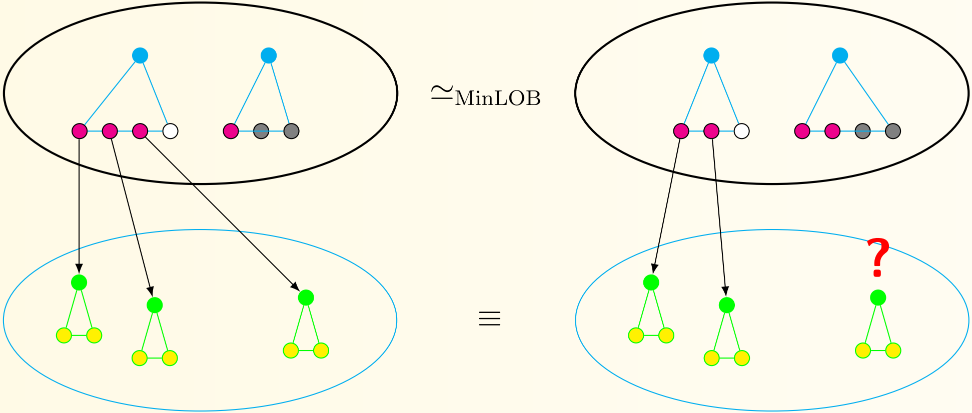

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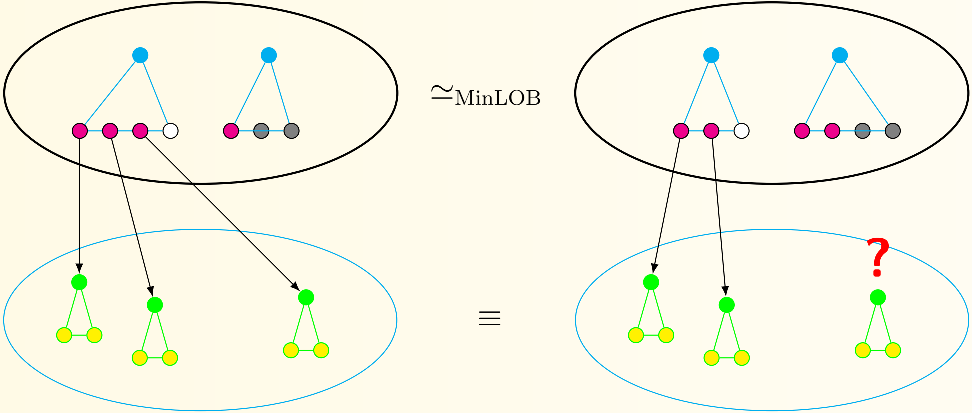

Any resolution? Simply ignore this inconsistency in dyn. algorithm...

- Pretend like if active labels do "not disappear" from particular tree, until all these labels are gone from the whole graph.

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Tweaking the dynamic algorithm

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$\Longrightarrow$ There is an XP algorithm for the MinLOB problem on digraphs of bounded clique-width, ... but it does not fit into the dynamic iterative Myhill–Nerode-based scheme. Why?

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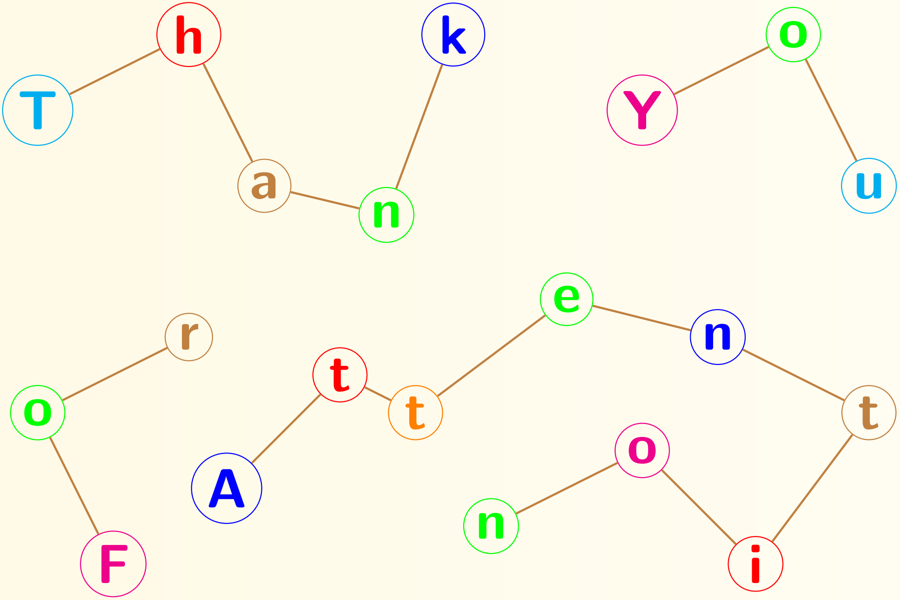
So, which one is more likely to be true?

- One can always find an (asymptotically?) optimal canonical partition consistent with $\otimes$.
- Or, there is something mysterious going on with Myhill–Nerode for XP algorithms.


Thank
You
for
Attention